

Rotating convection dynamics in Earth's outer core

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1. Introduction

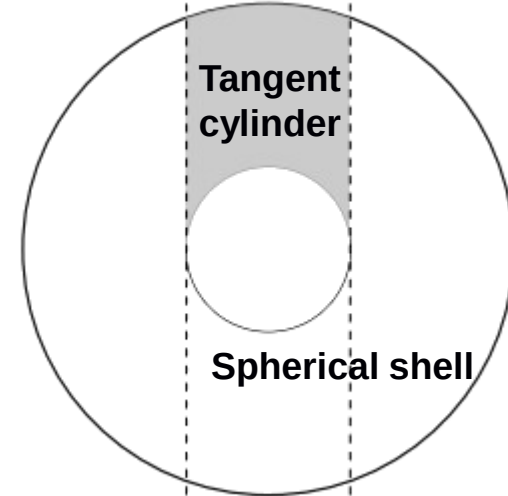
Flows of electrically conducting molten metal in the Earth's outer core have produced and sustained the Earth's magnetic field for much of our planet's history. This project aims to examine the dynamics of rotating convection and their regional variation in a spherical shell geometry. **The goal is to obtain polar-region scalings of parameters characterising heat transfer and compare them to global average results.** This will aid in determining the relevance of the results of experiments in cylindrical vessels to conditions in the Earth's core. The ultimate aim is to quantify the contribution of these flows to the creation of the Earth's magnetic field.

2. Physical Model and Governing Equations

- Original simulations in a spherical shell geometry, using the Boussinesq approximation
- Boundary conditions: no slip, fixed flux on each boundary
- Non-dimensionalised using the shell thickness h , the thermal diffusion time and the imposed heat flux

$$\frac{Ek}{Pr} \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right) + \Omega \times \mathbf{u} = -\nabla \tilde{P} + Ra T' \mathbf{r} + Ek \nabla^2 \mathbf{u}$$

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = \nabla^2 T, \quad \nabla \cdot \mathbf{u} = 0$$



- Input parameters: Ekman number, Prandtl number, Rayleigh number and radius ratio

$$Ek = \frac{\nu}{2\Omega h^2} \quad Pr = \frac{\nu}{\kappa} \quad Ra = \frac{\alpha g_0 \Delta T' h^3}{\nu \kappa} \quad \frac{r_i}{r_o} = 0.35$$

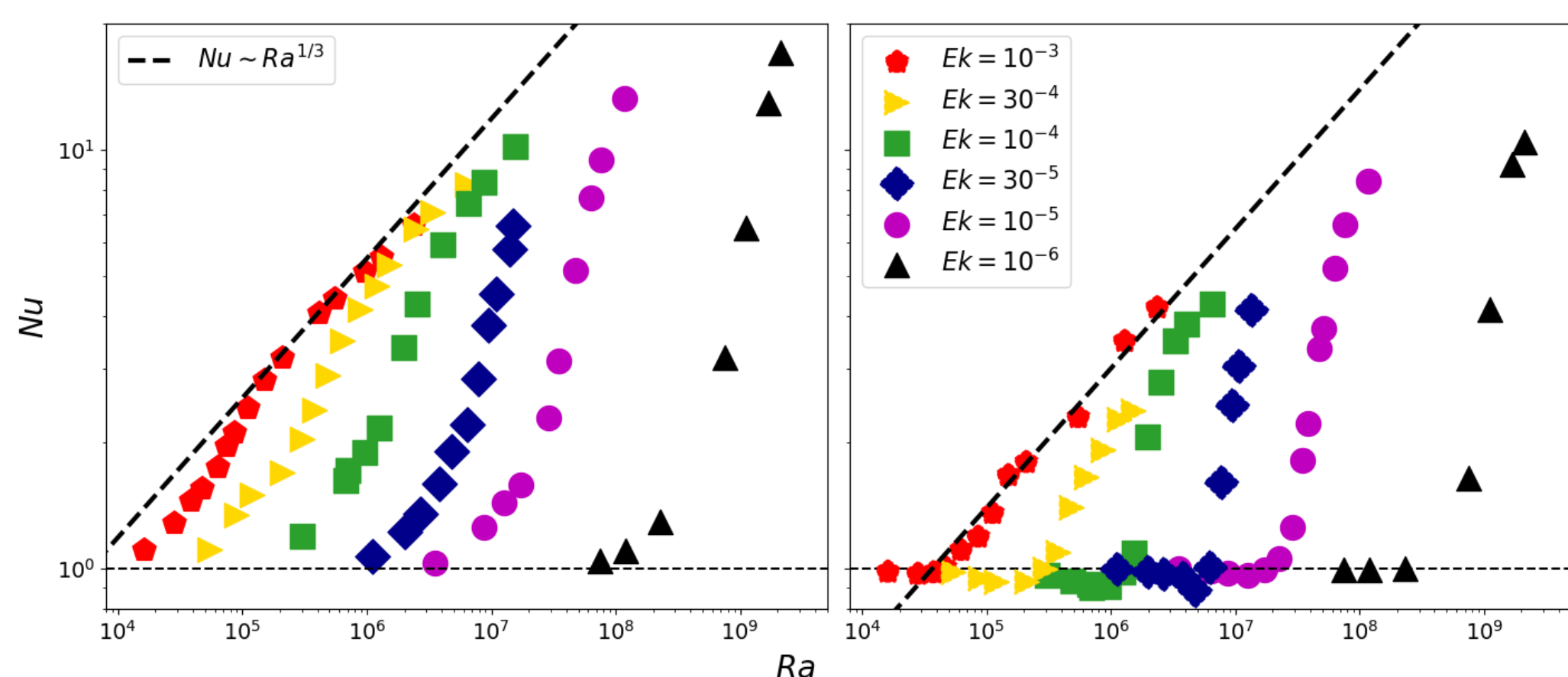
4. Modelling Challenges

When considering Earth's core, we must account for enormous variations in length and time-scales, giving us extreme parameter values. In the Earth's outer core, we expect $Ra \approx 10^{25}$ and $Ek \approx 10^{-15}$.

Simulations using cylinders with constant-curvature boundaries are more computationally efficient [6] and can also be compared directly with experimental results.

5. Heat transfer in shell and TC

Data was extracted from spherical shell simulations [1] with $Pr = 1$, $10^{-3} \leq Ek \leq 10^{-6}$ and $10^4 \leq Ra \leq 10^9$ and interpolated onto cylindrical grids. The Rayleigh number (a measure of the convective driving) was plotted against the global Nusselt number (quantifying heat transfer by convection) in the full shell (left, below) and compared to the local Nusselt number for the tangent cylinder region only (right, below).



In the spherical shell, we see at least three heat transfer scaling regimes [3]:

- Weakly non-linear: $Nu - 1 \sim (Ra/Ra_c) - 1$
- Rotation dominated: $Nu \sim Ra^{3/2} Ek^2$
- Rotation-independent: $Nu \sim Ra^{1/3}$, $Re \sim Nu^2$

where Ra_c is the critical Rayleigh number for the onset of convection. The dotted line shows the rotation-independent regime to which simulations at each Ekman number are expected to converge. These regimes in the TC are examined in box 6.

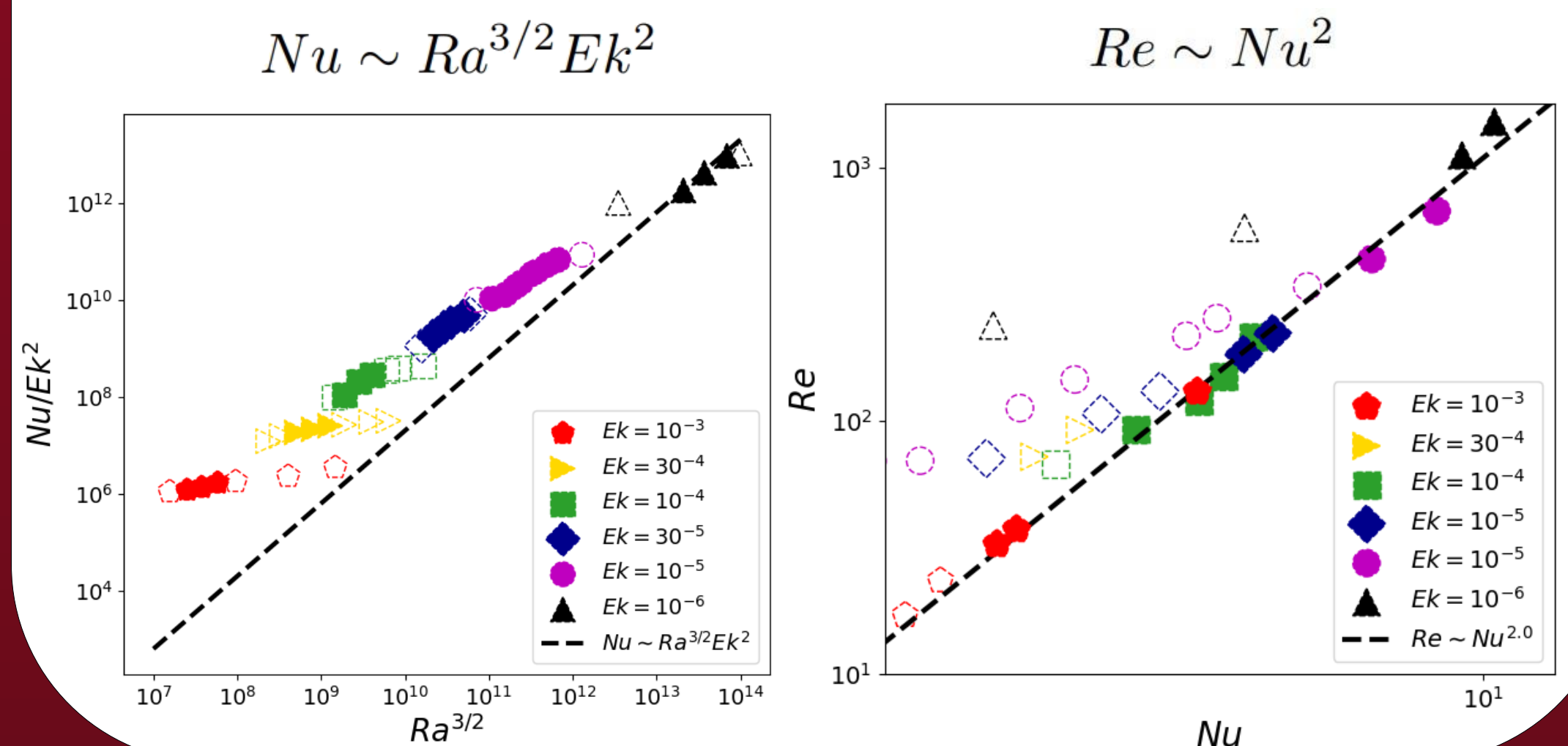
The TC has steeper scaling in the rotation dominated regime. In both geometries, the transition to rotation-independent scaling, governed by the Ekman number, may also depend on the interactions of the thermal and kinetic boundary layers (box 7).

3. The Tangent Cylinder (TC)

Flow dynamics alter by latitude in the spherical shell and there is a significant change at the boundaries of the Tangent Cylinder, the cylinder with boundaries circumscribing the inner core. **It is possible that these dynamics may be approximated by the plane-layer model of Rayleigh-Benard convection, where gravity and the rotation axis are aligned.**

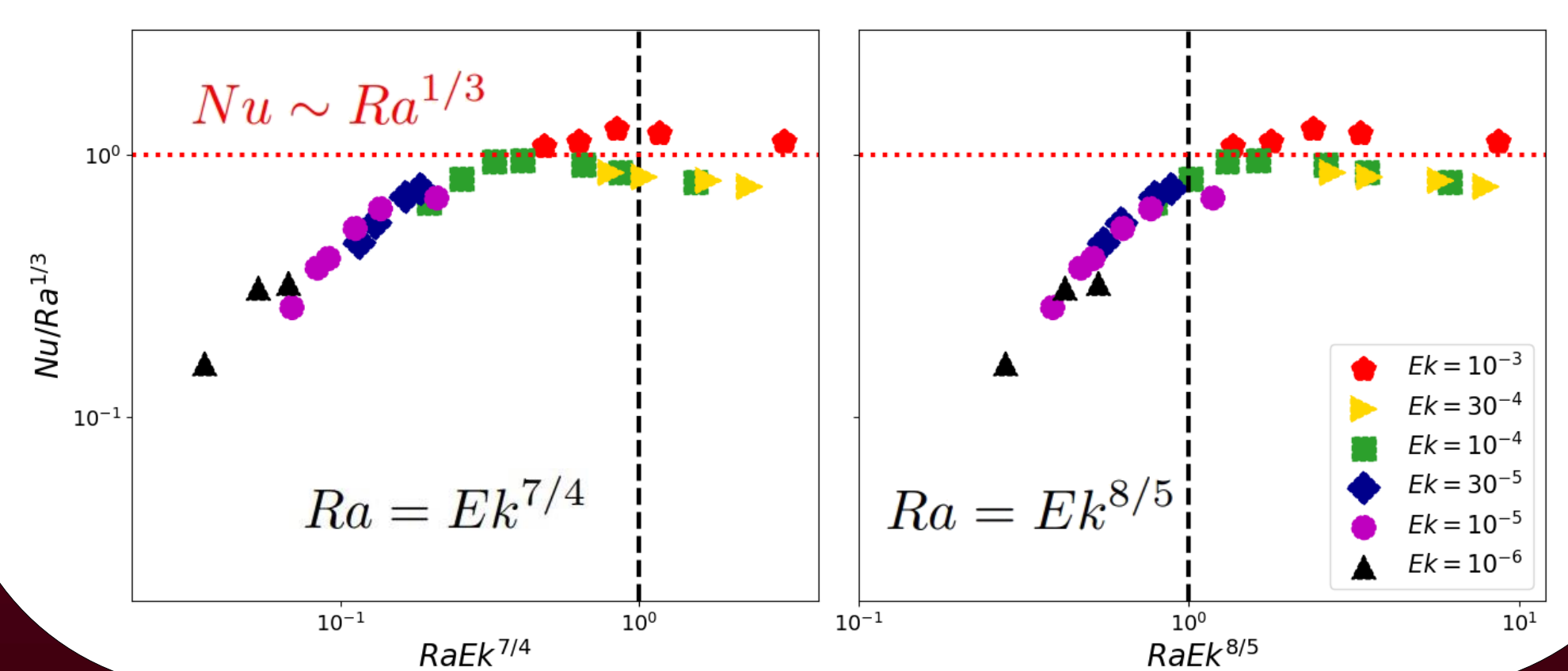
6. Scaling regimes in the TC

In the TC, the first regime is almost imperceptible. The second is approached as the Ekman number is reduced, (below left, showing all steeply scaled points as solid shapes). Below right: the Reynolds number plotted against the highest Nusselt numbers for each Ekman number in the TC (solid shapes) to show the transition to rotation-independent flow scaling.



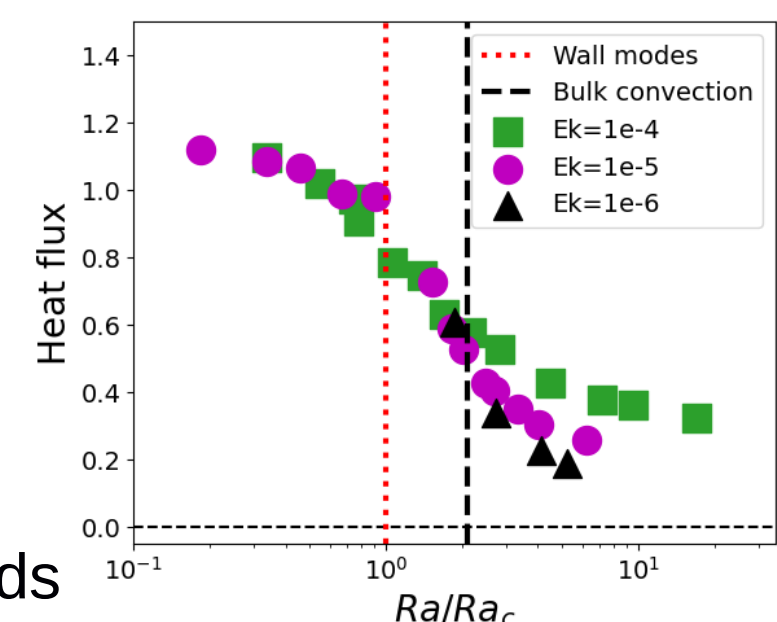
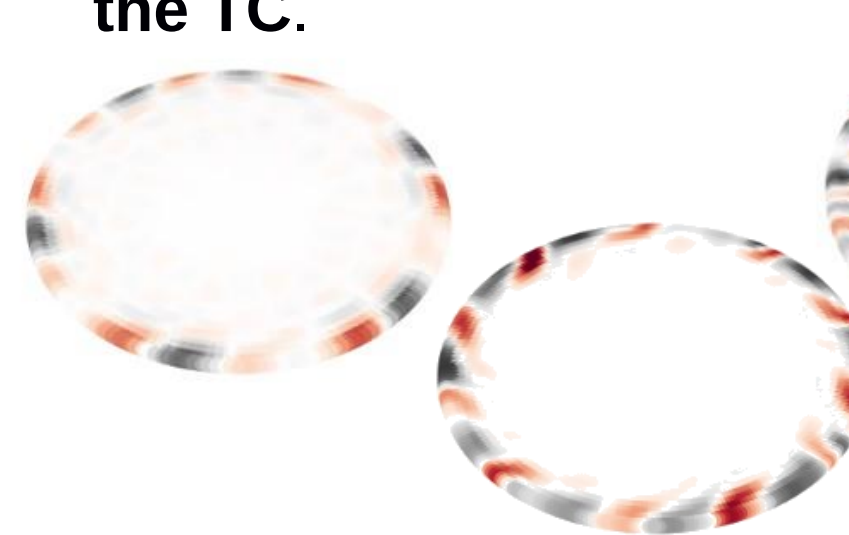
7. Transitions

The plots below show two possible points of transition to rotation-independent scaling in the TC. Left: when the thermal boundary layer becomes nested within the kinetic layer at $Ra = Ek^{7/4}$ [7]. Right: when the thermal boundary layer loses geostrophic balance at $Ra = Ek^{8/5}$ [9].



8. Future work

- Investigation of the influence of the sidewalls on flow and heat transfer.
- Reproduction of TC flow dynamics in a cylindrical simulation in Nek5000, using boundary conditions informed by the analysis of the TC.**



Right: Heat flux on the sidewall boundary tends towards an insulating condition at high Ra and low Ek . Above: modes at TC sidewall boundary (radial velocity).

References:

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- [9] Julien et al., Phys. Rev. Lett. 109 (25)

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