# Super-Resolution of Sparse Spatial-Observations of Navier Stokes

## A Physics-Informed Convolutional Neural Network Approach

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# Outline of Talk

## Contents and Preprint

Contents

1. Overview of the problem.
2. Quick recap on dynamical systems and introduce the Kolmogorov flow.
3. Demonstrate our methodology.
4. Showcase results.
5. Conclusion.

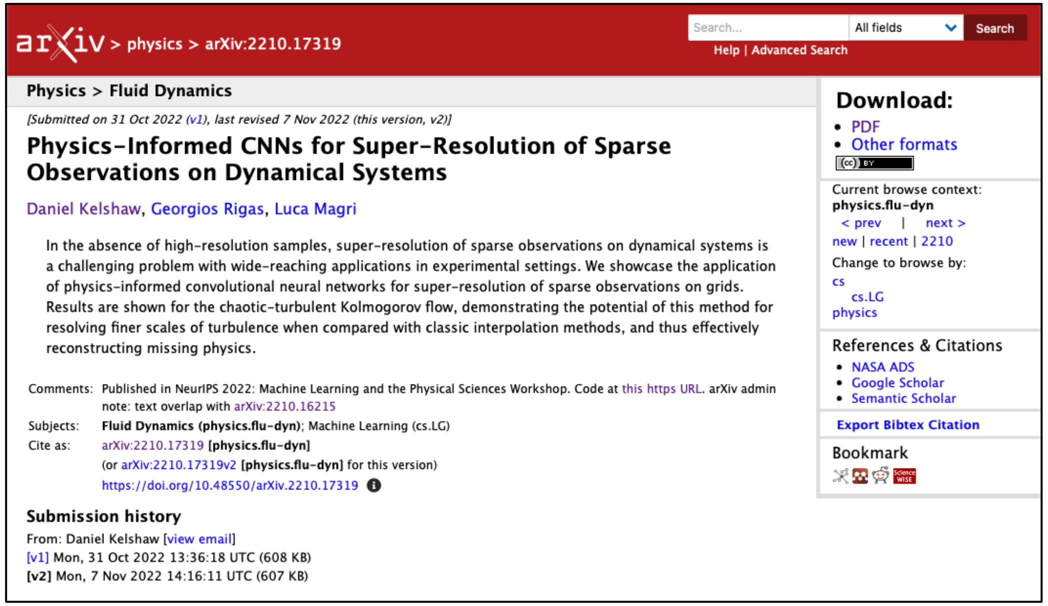
Preprint Available

Preprint available on arXiv →

**https://arxiv.org/abs/2210.17319**

Code available on GitHub

**https://github.com/magrilab/pisr**



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*[Submitted on 31 Oct 2022 (v1), last revised 7 Nov 2022 (this version, v2)]*

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Subjects: Fluid Dynamics (physics.flu-dyn); Machine Learning (cs.LG)

Cite as: arXiv:2210.17319 [physics.flu-dyn]
(or arXiv:2210.17319v2 [physics.flu-dyn] for this version)
https://doi.org/10.48550/arXiv.2210.17319

**Submission history**

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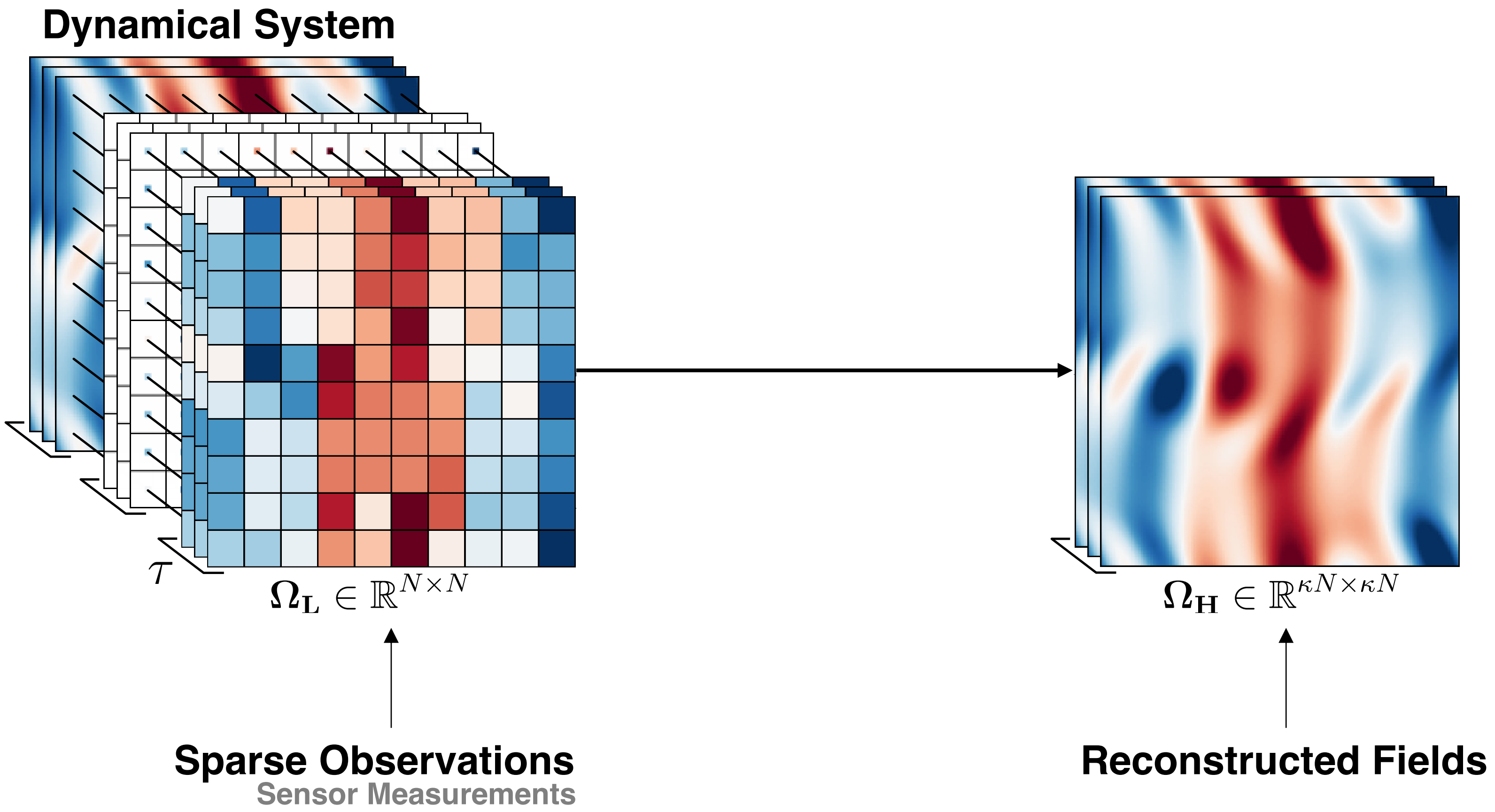
D. Kelshaw, G. Rigas, and L. Magri, "Physics-Informed CNNs for Super-Resolution of Sparse Observations on Dynamical Systems," in *NeurIPS 2022 Workshop on Machine Learning and the Physical Sciences, 2022*, **https://arxiv.org/abs/2210.16215**

# What Problem are we Tackling?

Super-resolution **with no examples**

The Super-Resolution Problem




**How can we learn to super-resolve sparse observations with no high-resolution examples?**

# The Dynamical System

## Kolmogorov Flow

**Prototypical Dynamical System / Residual**

$$\partial_t \boldsymbol{u} - \mathcal{N}(\boldsymbol{u}; \lambda) = 0$$

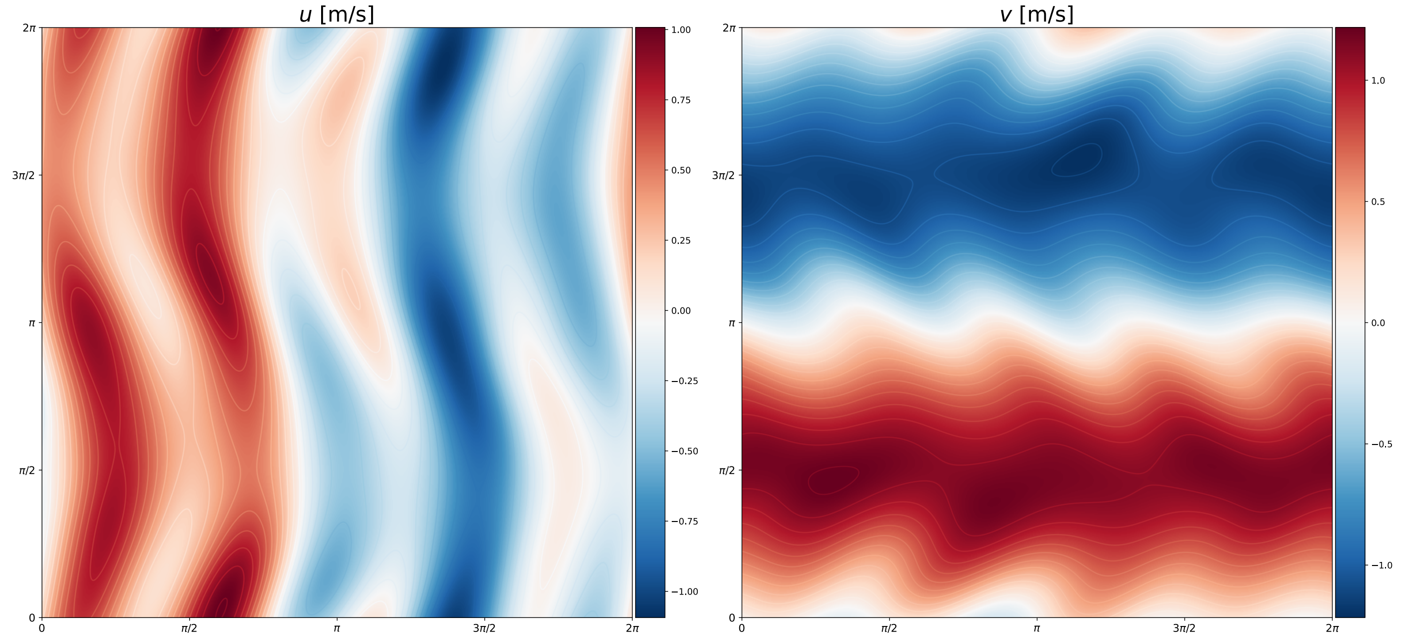
$$\mathcal{R}(\boldsymbol{u}, \lambda) \equiv \partial_t \boldsymbol{u} - \mathcal{N}(\boldsymbol{u}; \lambda)$$

**The Kolmogorov Flow**

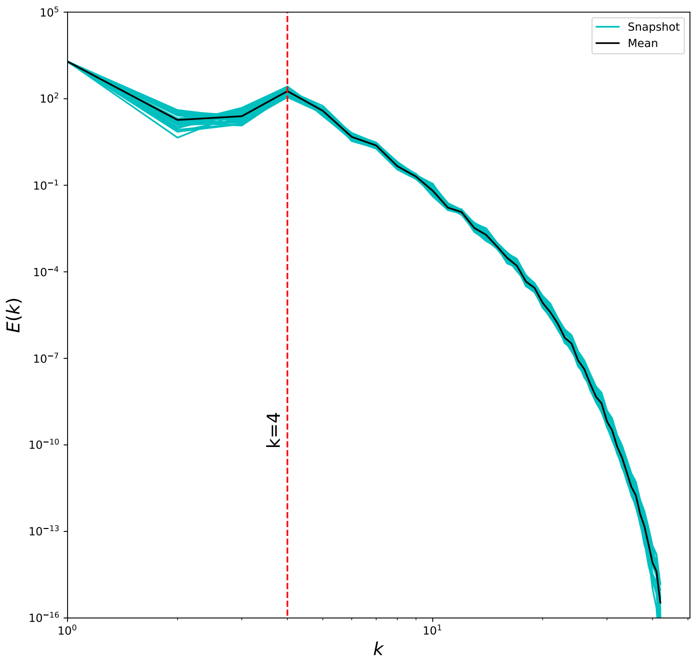
**Navier-Stokes:** 2D Incompressible
Periodic spatial boundary conditions on $\Omega \in [0, 2\pi) \subset \mathbb{R}^2$
Periodic forcing in a single-direction.

$$\partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \nu \Delta \boldsymbol{u} + g(\boldsymbol{x})$$

**Sample Fields**




**Energy Spectrum :: RE = 34**



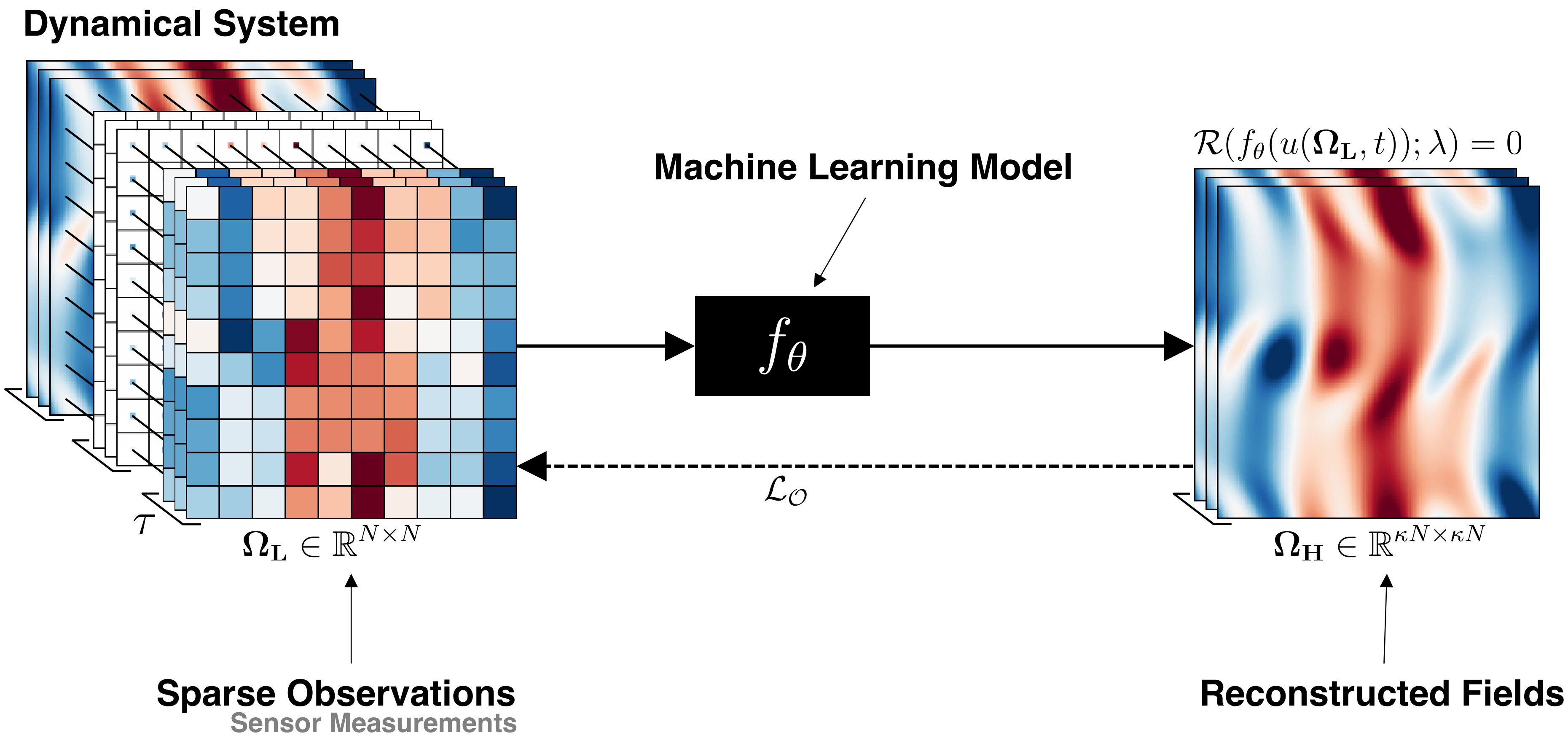
# Our Approach for Super-Resolution

## Physics-Informed Convolutional Neural Network

Introducing the Model

Our goal is to find a function $f_\theta$ capable of mapping the low-resolution field to the high-resolution field.

$$f_\theta : u(\boldsymbol{\Omega}_{\boldsymbol{L}}, t) \rightarrow u(\boldsymbol{\Omega}_{\boldsymbol{H}}, t)$$

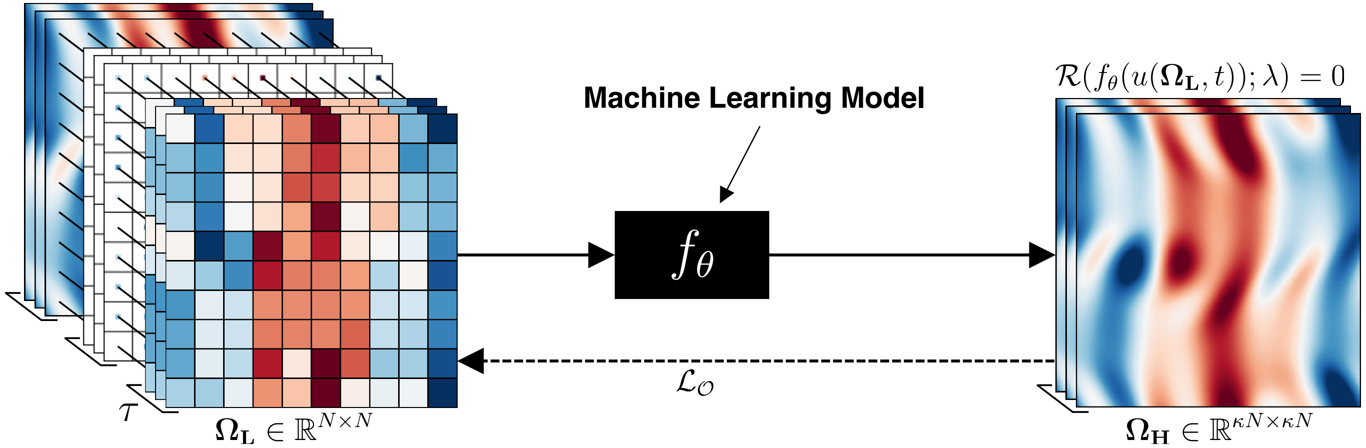


# Methodology

## Defining the Loss

The Task

$$f_\theta : u(\boldsymbol{\Omega_L}, t) \rightarrow u(\boldsymbol{\Omega_H}, t)$$




Setting up the Optimisation Problem

$$\theta^* = \underset{\theta}{\operatorname{argmin}}\, \mathcal{L}_\theta$$

$$\mathcal{L}_\theta = \mathcal{L}_\mathcal{O} + \alpha \mathcal{L}_\mathcal{R}$$

**Minimise the error of predictions at sensor locations**

**Minimise the residual of the predictions**

More on the Residual Loss

Differentiable pseudospectral discretisation for the differential operator:

$$s : \hat{u}(\boldsymbol{k}, t) \rightarrow \partial_t \hat{u}(\boldsymbol{k}, t)$$

1. Euler forward-difference to approximate derivative.
2. Compare with analytical derivative from the solver.

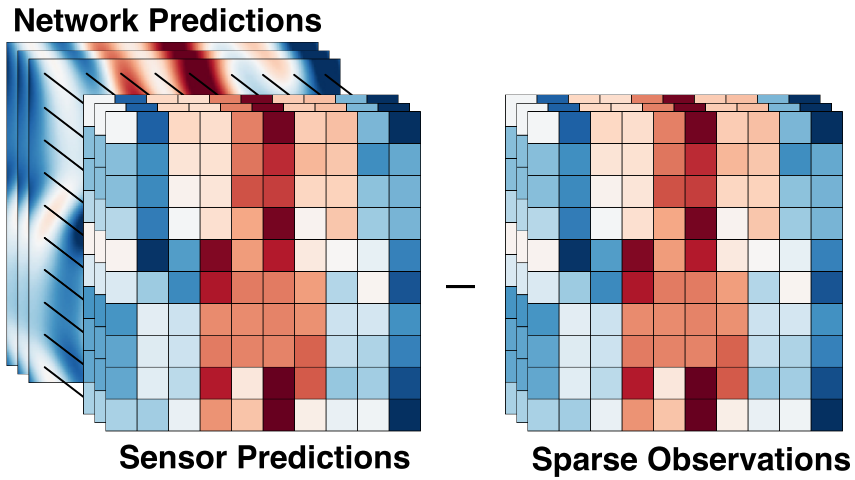
**Key Point:** This allows us to embed knowledge on the dynamical system in the loss *a priori.*

# Methodology

## Observation-Based Loss

#### Defining the Observation-Based Loss

1. Utilise sparse measurements effectively.
2. Minimise error between model predictions and observations.




$$\mathcal{L}_{\mathcal{O}} = \frac{1}{|\mathcal{T}|}\sum_{t\in\mathcal{T}} \left\| \quad - \quad \right\|^2_{\Omega_L}$$

# Methodology

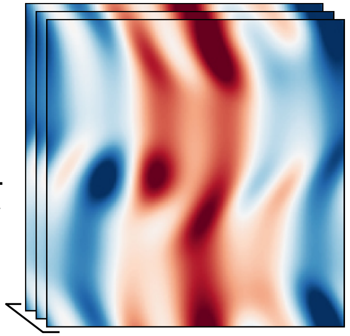
## Residual-Based Loss

**Defining the Observation-Based Loss**

1. Utilise knowledge of the physical system.
2. Minimise residual of network predictions.

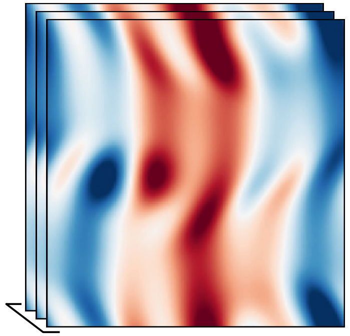
$$\mathcal{L}_{\mathcal{R}} = \frac{1}{|\mathcal{T}|}\sum_{t\in\mathcal{T}} \Bigg\| \frac{\partial}{\partial t}$$

Network Predictions



$$- \quad \mathcal{N}\Bigg($$

Network Predictions

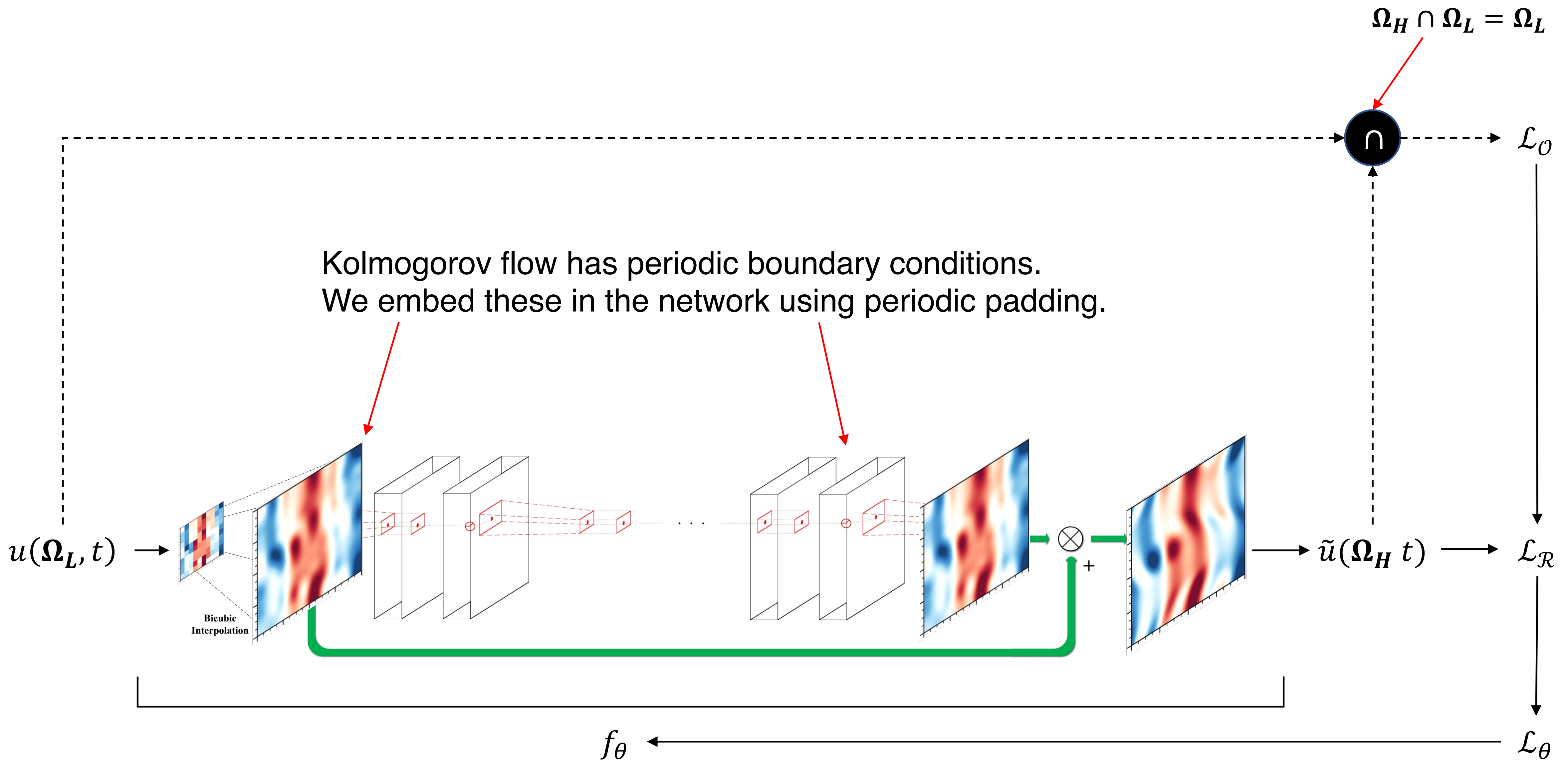


$$, \lambda \Bigg) \Bigg\|_{\boldsymbol{\Omega_H}}^{2}$$

# Physics-Informed Convolutional Neural Network

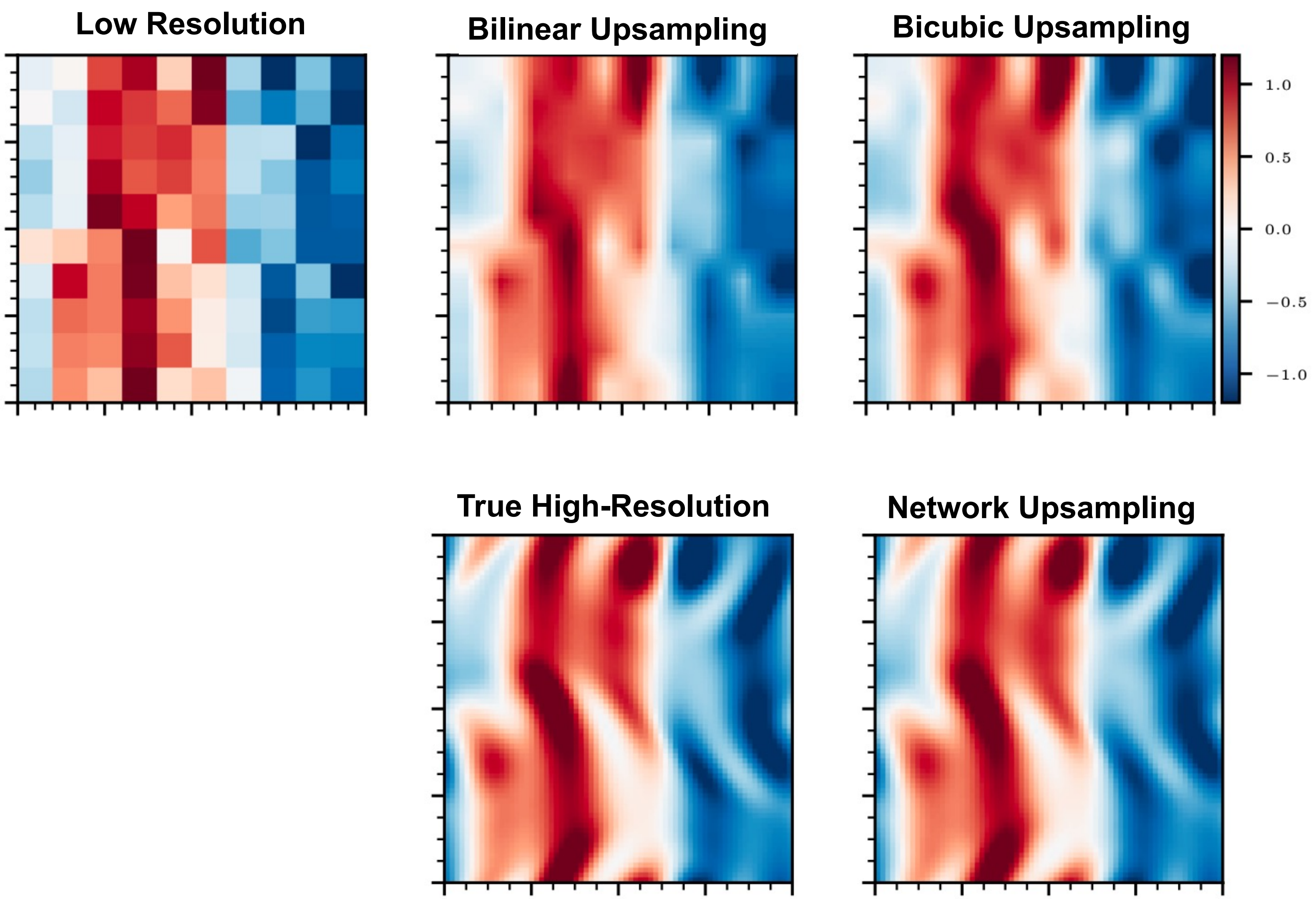
## The Architecture

Kolmogorov flow has periodic boundary conditions.
We embed these in the network using periodic padding.



# Results – We can retrieve Navier Stokes solution.

**Single Flow Field Prediction – Comparison with Naïve Upsampling.**

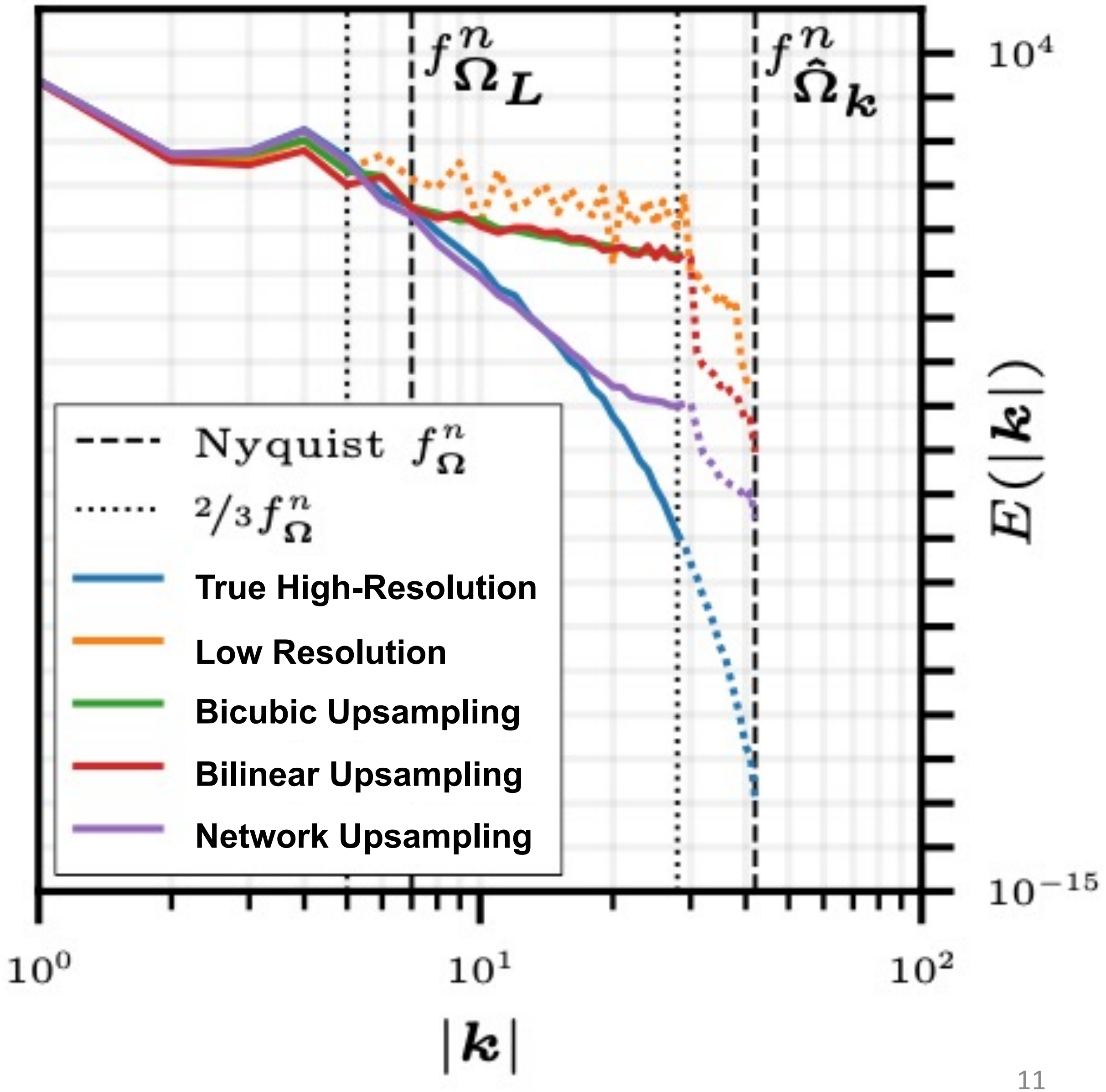


# Results – Looking at the Energy Spectrum

**Recovering Underlying Physics / Anti-aliasing.**

**Recovering Underlying Physics**

- Upsampled results are subject to high degrees of aliasing
- Network predictions recover smaller scales of turbulence – a result of the physics-informed loss.



# Conclusions

What we accomplished & what's next?

Our accomplishments:

1. Produced physics-informed convolutional neural network capable of super-resolution.
2. Embedded knowledge about the boundary conditions in the network.
3. Demonstrated physically-principled results which generalize across the entire time-domain.

What's next?:

Looking at the effect of noisy low-resolution samples.

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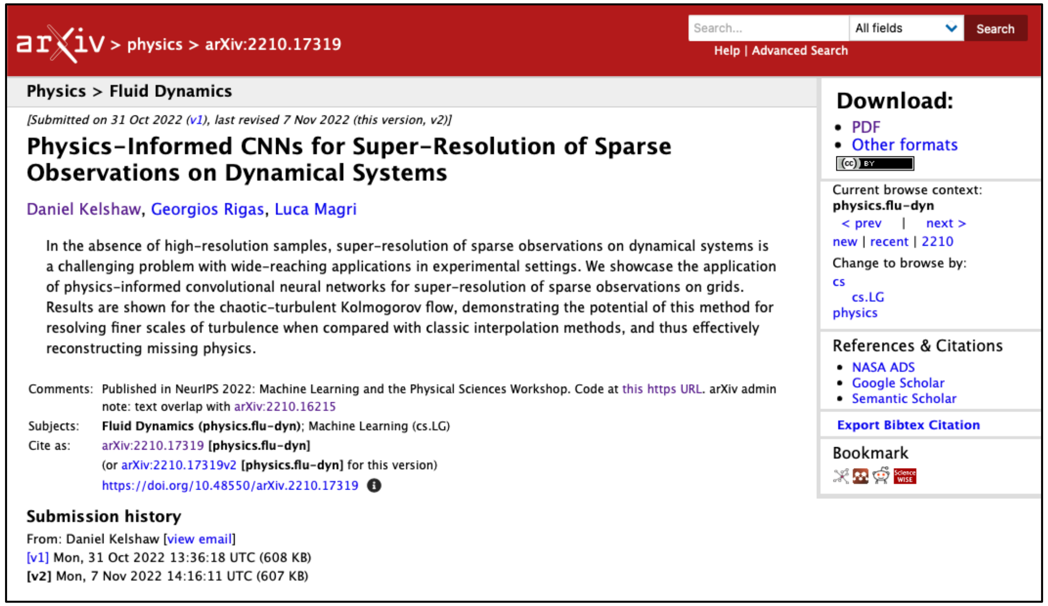

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