UNIVERSITY OF CAMBRIDGE

Inverse problems in fluid mechanics for enhanced MR velocimetry

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Acknowledgement

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CUED, Cambridge

Experimental data (flow-MRI)

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MRRC, Cambridge

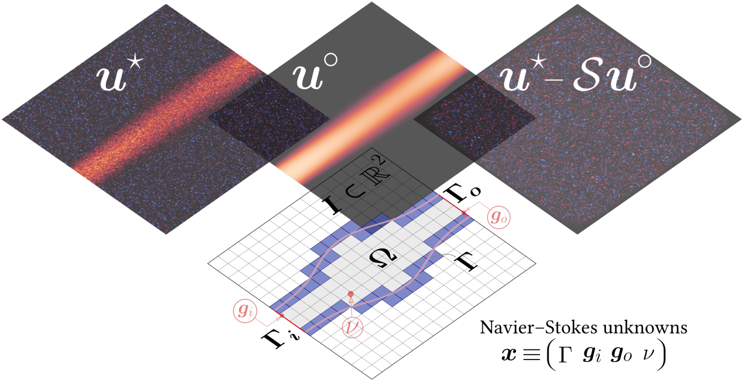
Outline

I. Bayesian inverse N–S problem for *full signals*
II. Physics-informed compressed sensing for *sparse signals*
III. Reconstruction of 3D aortic flow
IV. Conclusions

Part · I

Bayesian inverse N–S problem for *full signals*

Bayesian inverse N–S: a digital twin for flow imaging



$\boldsymbol{u}^\star$: noisy velocity image

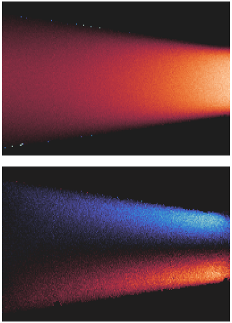
$\boldsymbol{u}^\circ$: N–S reconstruction (digital twin)

$\boldsymbol{u}^\star - \mathcal{S}\boldsymbol{u}^\circ$: filtered noise & artefacts

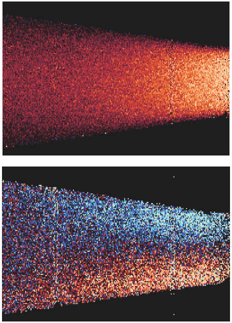
$\boldsymbol{x}$: most likely (inferred) Navier–Stokes parameters

A. Kontogiannis *et al.*, *J. Fluid Mech.*, 944(A40), 2022.

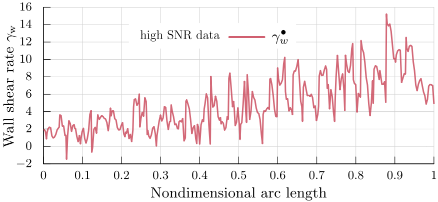
Denoising and improved wall-shear rate estimation



High SNR
PC-MRI images
∼5hrs



Low SNR
PC-MRI images
∼10min (×30 faster)

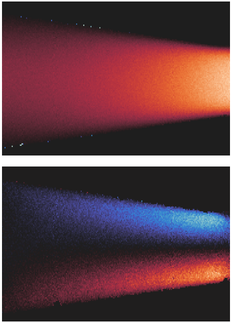


Even with high SNR data
wall shear stress is
badly approximated

- **Addresses** two major **shortcomings of flow-MRI**:
  - i. noise increases as spatial resolution increases
  - ii. partial volume effect (irregularities) near the boundaries

  that hinder the accurate estimation of **wall shear stresses**.

Denoising and improved wall-shear rate estimation



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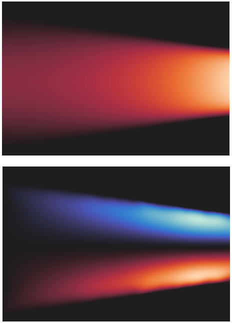
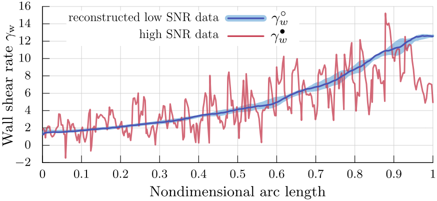


Reconstructed low SNR
PC-MRI images
$\sim$10min ($\times$30 faster)



Reconstructed low SNR data
approximate better the
wall shear stress

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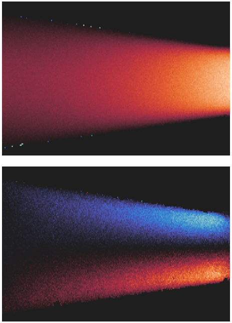

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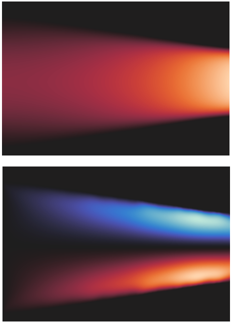

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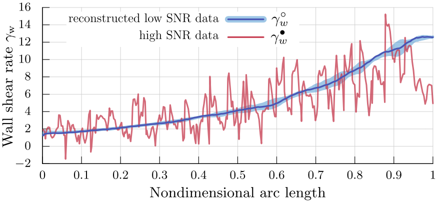

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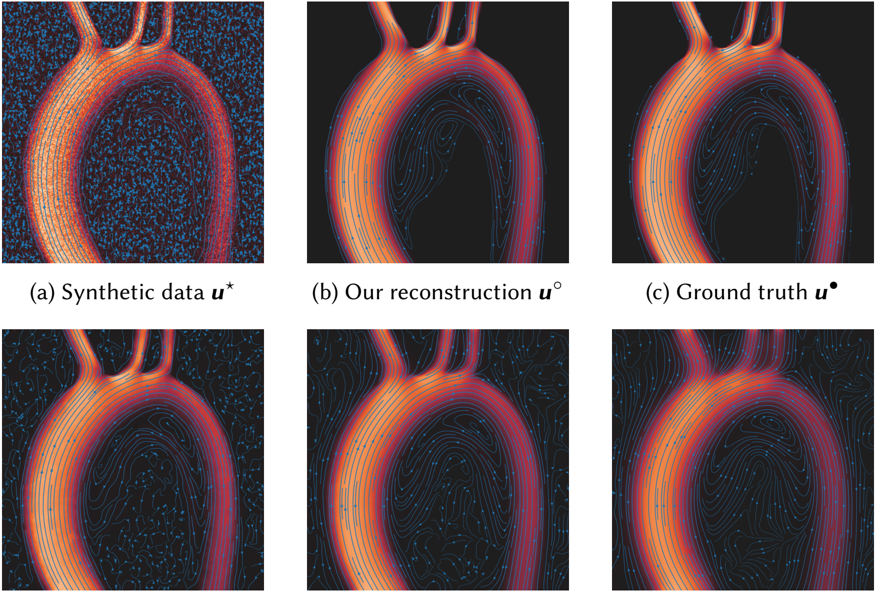


(a) Synthetic data $\boldsymbol{u}^{\star}$

(b) Our reconstruction $\boldsymbol{u}^{\circ}$

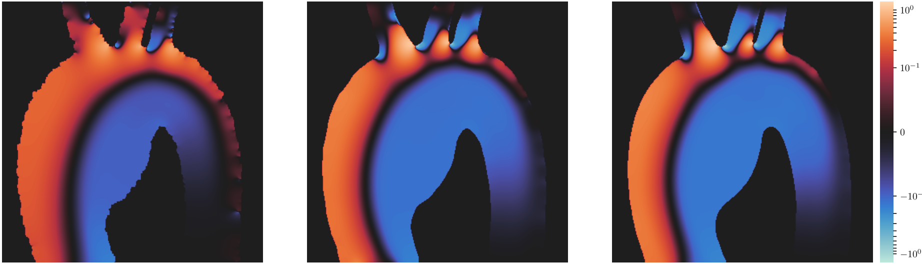
(c) Ground truth $\boldsymbol{u}^{\bullet}$

(d) TV-B $\lambda/\lambda_0 = 0.1$

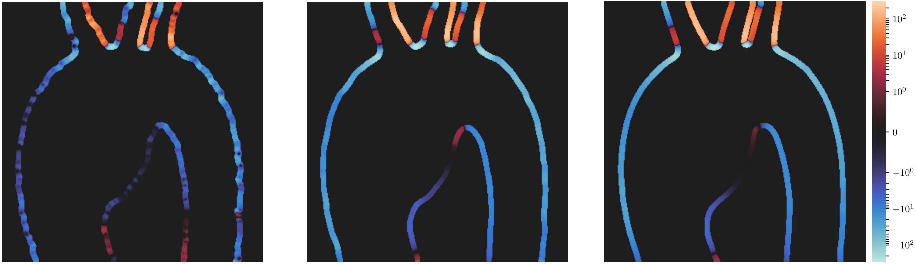
(e) TV-B $\lambda/\lambda_0 = 0.01$

(f) TV-B $\lambda/\lambda_0 = 0.001$

Figure: Streamlines in the simulated 2D model of an aortic aneurysm ($\mathrm{Re} = 500$).



(a) Zeroth iteration p_0 (b) Our reconstruction p° (c) Ground truth $p^\bullet$



(d) Zeroth iteration $(\gamma_w)_0$ (e) Our reconstruction γ_w° (f) Ground truth $\gamma_w^\bullet$

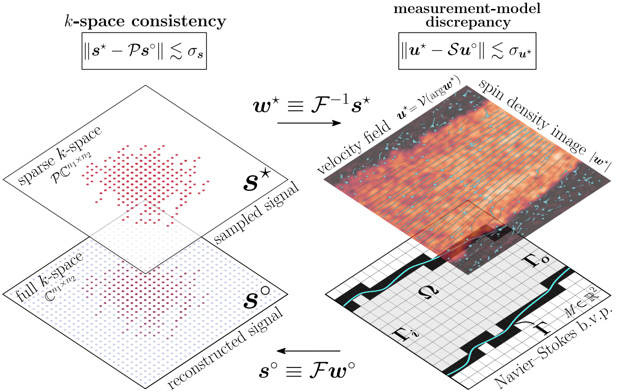
Figure: Inferred wall shear stress and pressure.

Part · II

Physics-informed compressed sensing (PICS)

for *sparse signals*

Physics-informed compressed sensing for flow-MRI



$s^\star$: sparsely-sampled k-space signal.

s° : reconstructed signal using a physics-aware digital twin.

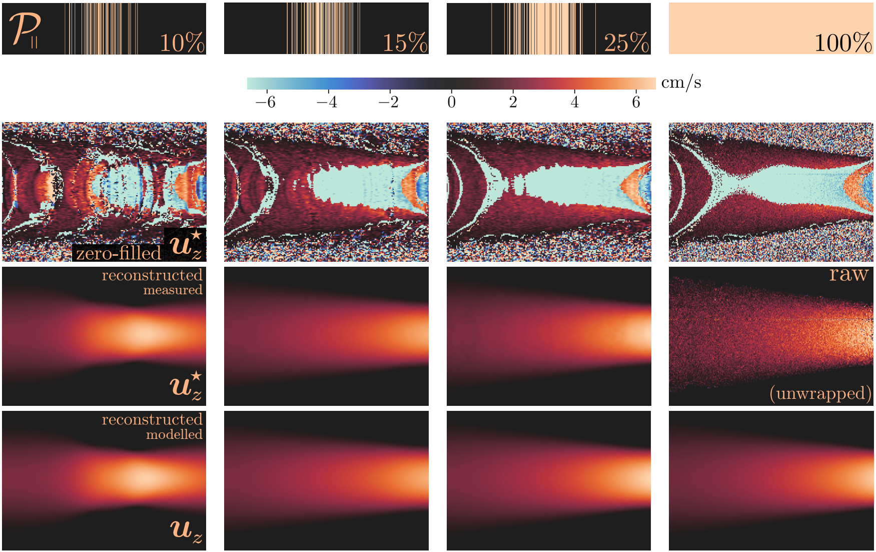
$u^\star$: reconstructed *measured* velocity.

u° : reconstructed *modelled* velocity.

Γ : most likely boundary of the object Ω.

A. Kontogiannis and M. Juniper, *IEEE Trans. Image Process.*, 32:281-294, 2023.

Reconstruction for barcode sparse sampling with $\mathcal{P}_{\parallel}$



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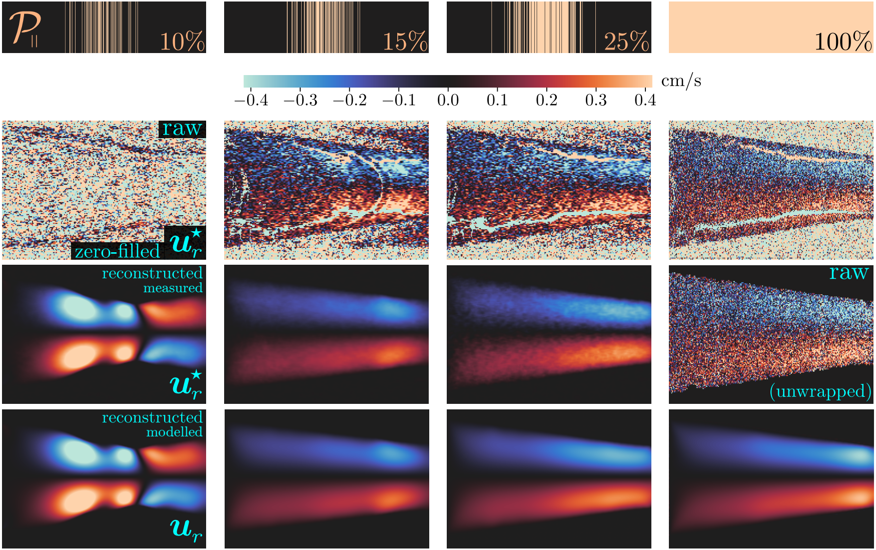


Shape inference, pressure, and wall shear stress

- Because PICS integrates a N–S model, it can furthermore infer the hydrodynamic pressure and the wall shear stresses.

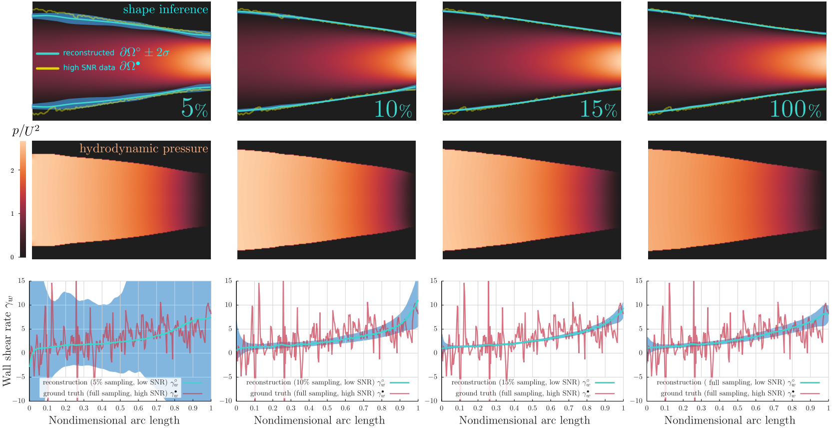


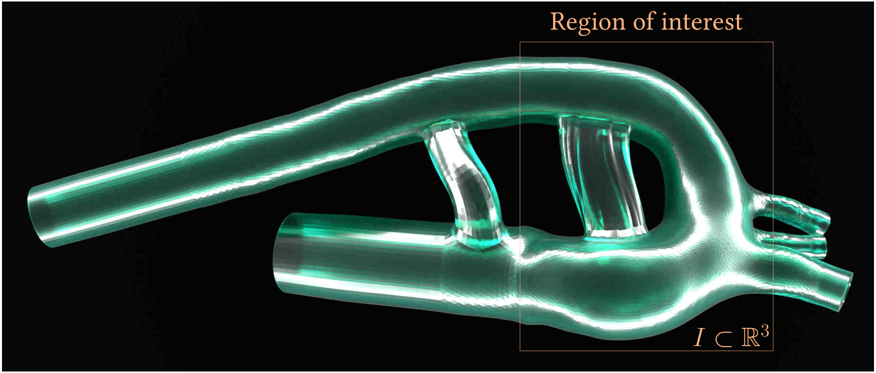
Figure: Inferred shape, $\partial\Omega^\circ$, and velocity magnitude (first row) for 5%, 10% and 15% $\mathcal{P}_\odot$-sampling, and 100%-sampling.

Part · III

Reconstruction of 3D aortic flow

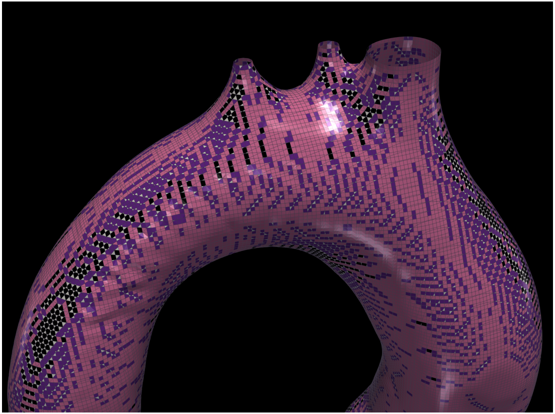
3D-printed physical model of an aortic arch

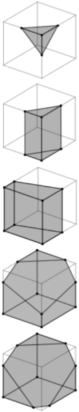
- Steady 3D flow-MRI experiments at $Re \simeq 500, 1000, 1500$
- Reconstruction of noisy data in the region of the aortic arch



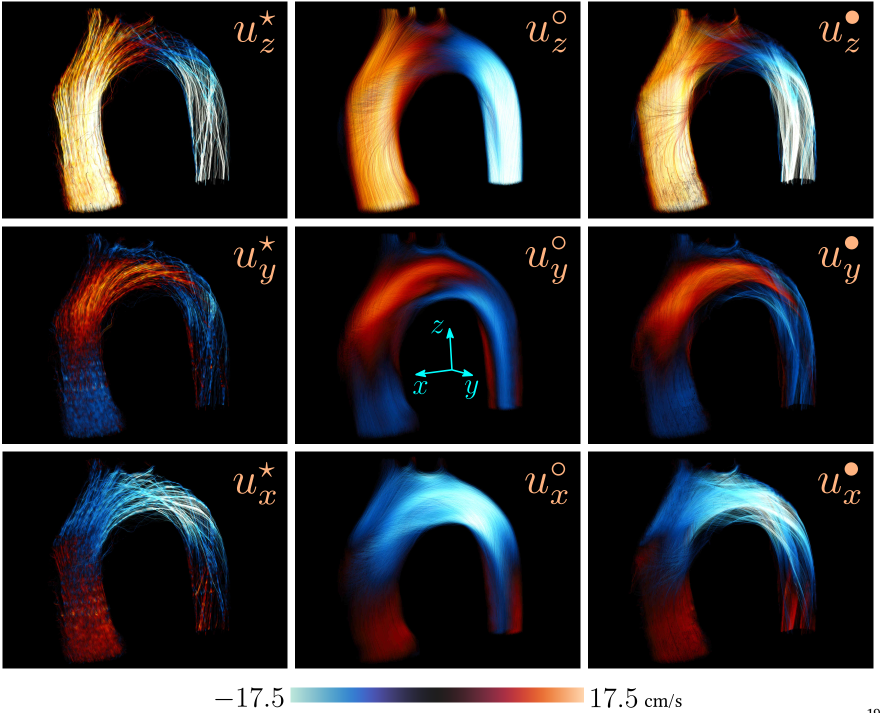
Cut-cell FEM (immersed boundary)

- Boundary is implicitly defined by the signed distance field
- MPI implementation in Python using PETSc



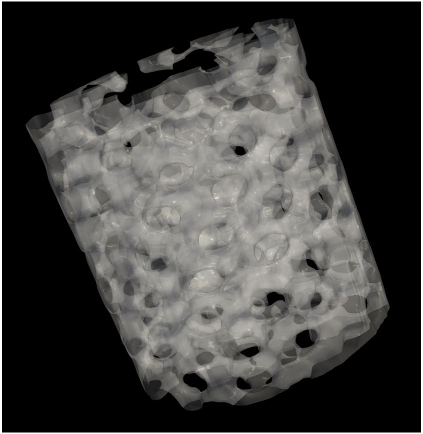


Based on *Massing, A. et. al. Comput Methods Appl Mech Eng, 328:262–300, 2018*.

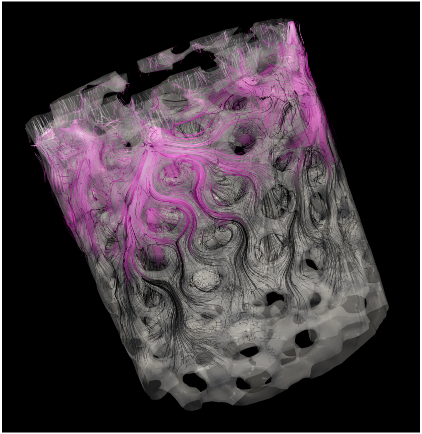


$u_z^\star$
u_z°
$u_z^\bullet$
$u_y^\star$
u_y°
$u_y^\bullet$
z
x
y
$u_x^\star$
u_x°
$u_x^\bullet$
-17.5
17.5 cm/s

3D porous media flow reconstruction - work in progress



(a) Segmented geometry of a packed bed



(b) Simulated flow through the packed bed

Figure: Demonstration of the parallel 3D Navier–Stokes (cut-cell finite element) solver that we have developed in a packed bed geometry.

Conclusions

Summary

- Formulated a Bayesian inverse N–S problem for the reconstr. of full MRV signals
- Extension to physics-informed compressed sensing (PICS) for sparse MRV signals

So what?

- Allows significant reductions in scanning time (up to $\sim$ x250 acceleration)
- Infers posterior mean and covariance of the geometry, pressure, WSS etc.
- Learns a digital twin of the MRV experiment paving the way to patient-specific cardiovascular modelling
- Can simulate different flow conditions using the learned digital twin
- Offers high explanatory power (explains much from less) and interpretability

What's next?

- Extend the implementation to periodic and unsteady flows
- Incorporate non-Newtonian fluid models and infer their parameters
- Apply algorithms to reconstruct flow-MRI data of *in-vivo* cardiovascular flows

Thank you!

Questions?