A reduced order model for space-time wave statistics with probabilistic decomposition-synthesis method

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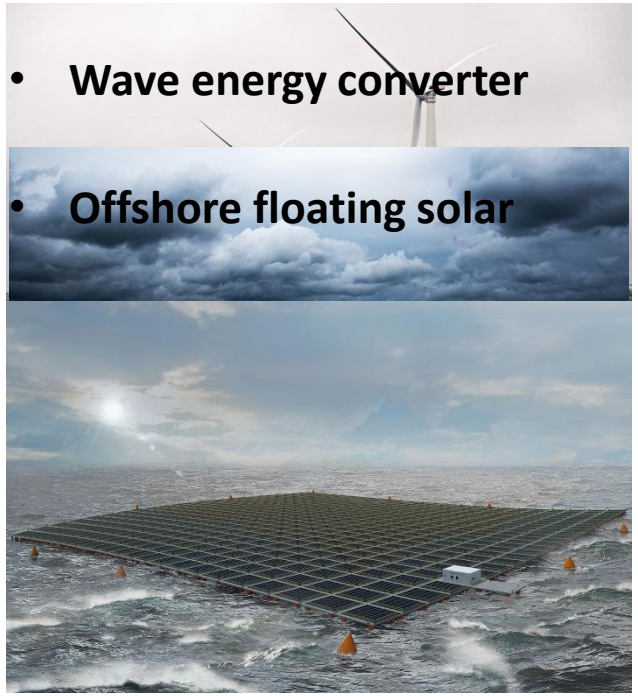
Supervised by Thomas Adcock
30/03/2023

Offshore Engineering

Cost of failure ≈ £2-20B

- Offshore wind
- Wave energy converter
- Offshore floating solar

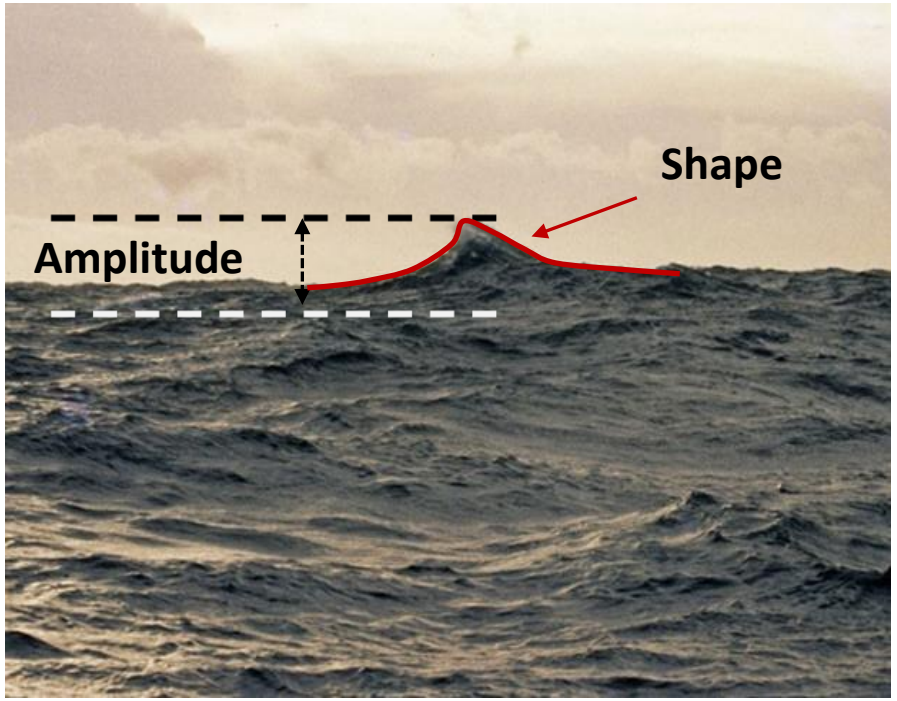


Other offshore engineering structures…

19/06/2022, BBC, Huge waves crash against swaying North Sea oil rig. This recording is to be used only for educational and non-commercial purposes under the terms of the ERA Licence

Objectives

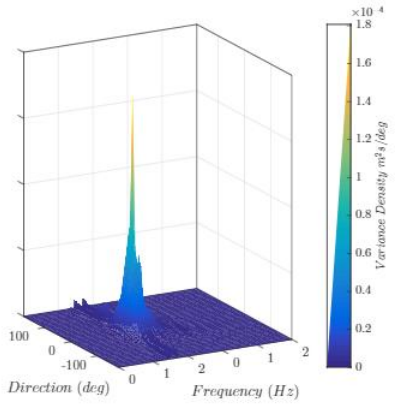
- Wave crest **amplitude** statistics
- Most probable **shape** of extreme waves
- **Physics-based data-driven models that are not ‘backbox’**

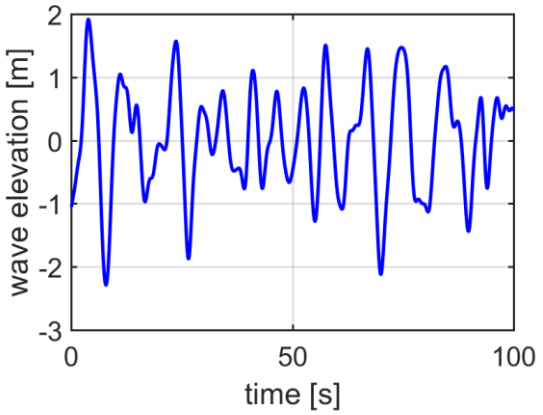


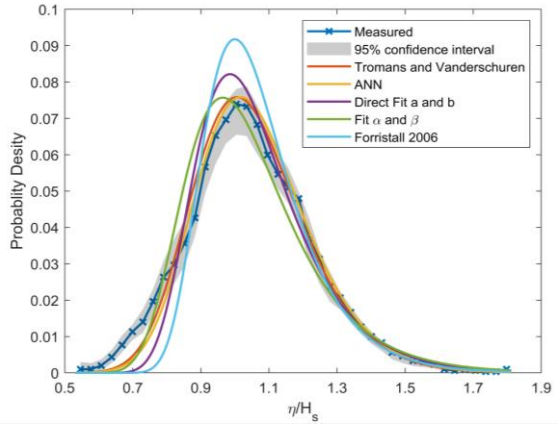
The Challenge

- Although with very little turbulence, nonlinear amplification is significant (up to 30% wave amplitude)
- Very long time scale – Several decades and large spatial scale – several kilometres
- Complexity in the nonlinear physics (Benjamin-Feir instability, wave breaking, wave-current interactions, wind-wave interactions…)
- Balance between in the numerical model accuracy and computation resources available

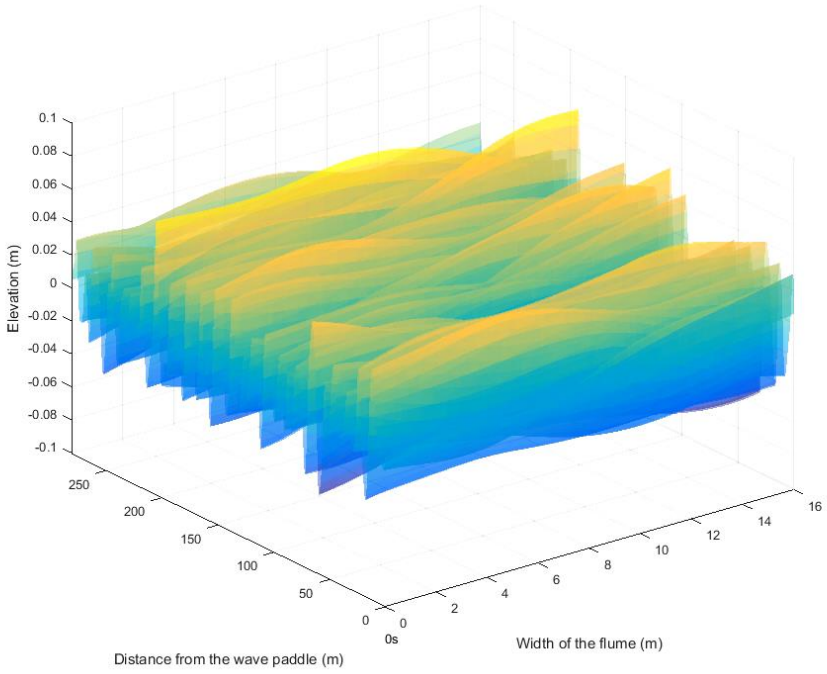
Wave spectrum → **Time series** → **Probability distribution**

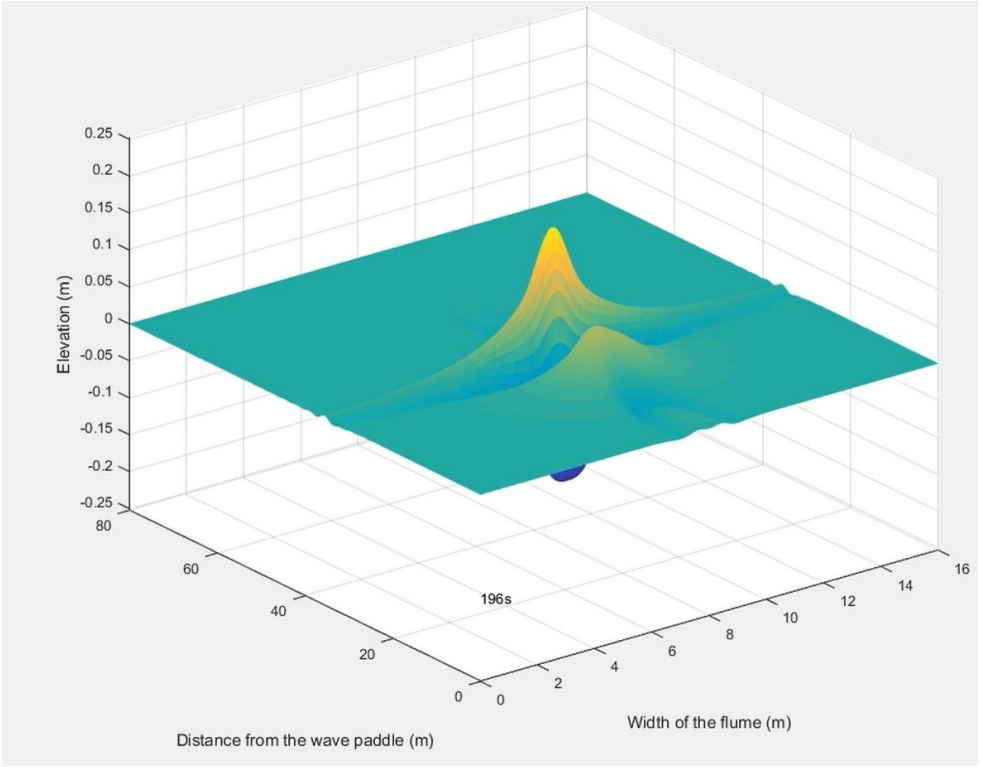




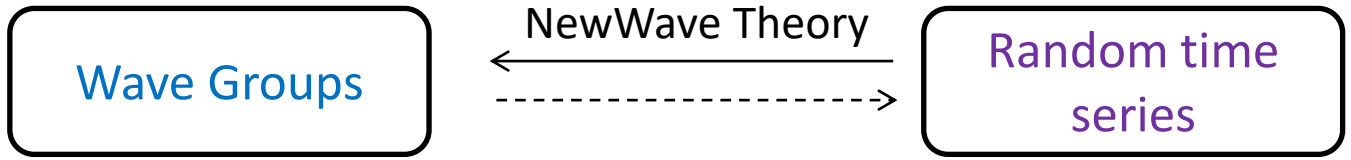


Random waves vs. Wave Group





A framework describing extremes



NewWave theory: the averaged shape of largest events can be described as NewWave groups under linear assumption.

- Can we isolate the extreme events observed in the random time series as individual wave groups?
- Can we predict the **nonlinear changes** of these extreme events observed in random time series with individual wave groups?
- Can we use these wave groups to predict **wave statistics** for random time series with individual wave groups?

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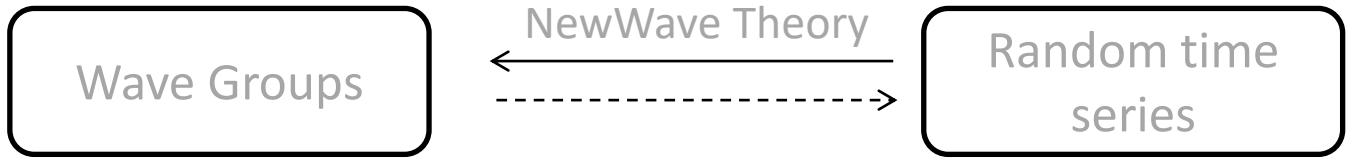

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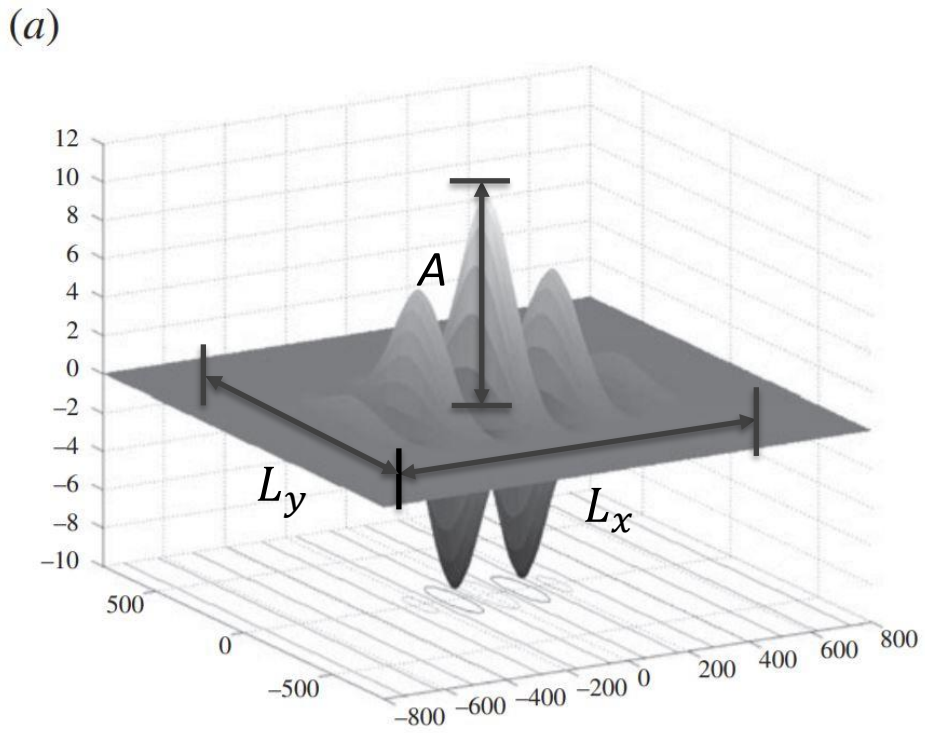
Gaussian wave groups



$$u_0(x,y) = A\exp\left[-\frac{x^2}{L_x^2} - \frac{y^2}{L_y^2}\right],$$

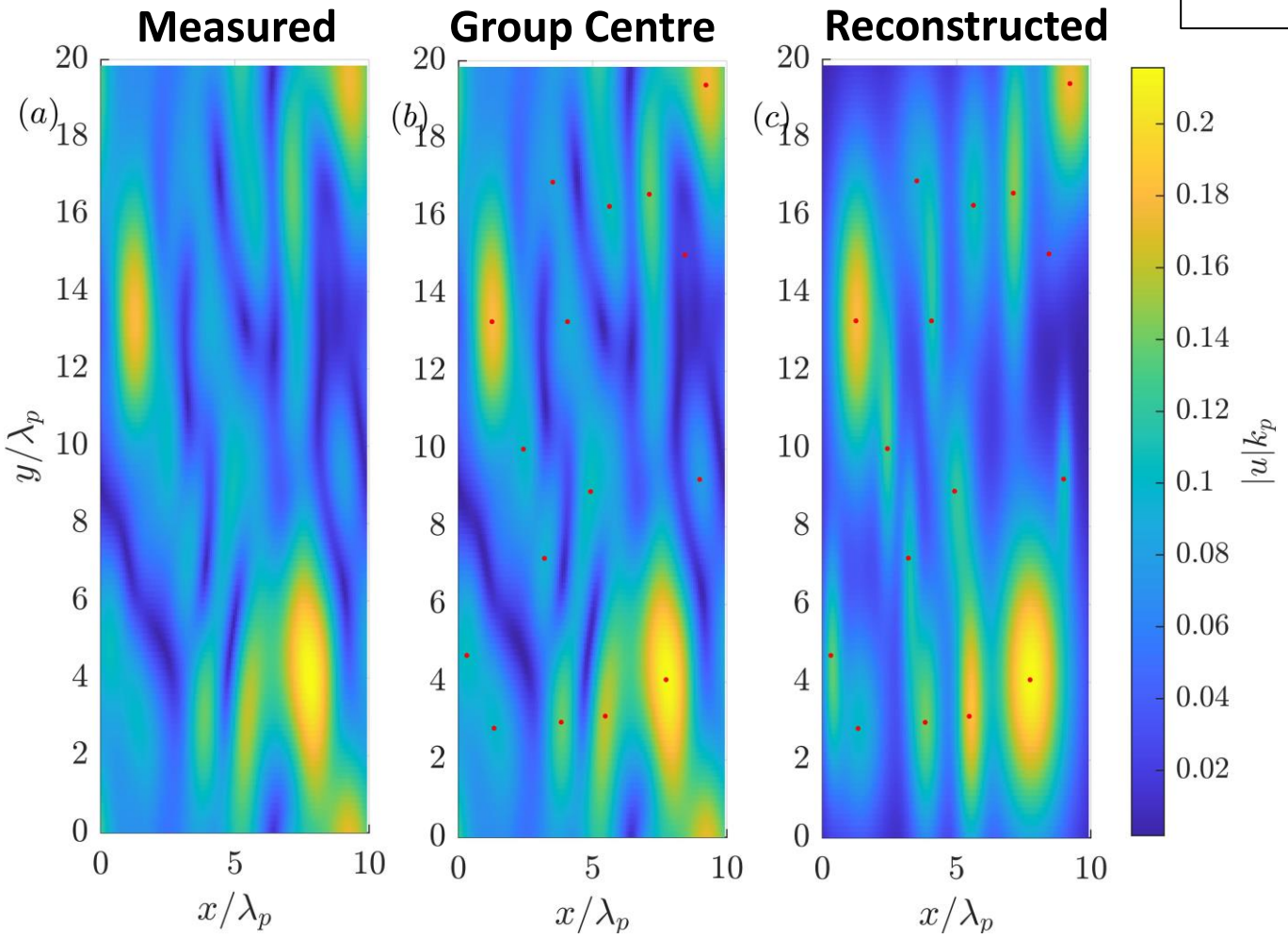
A: Envelope amplitude at linear focus

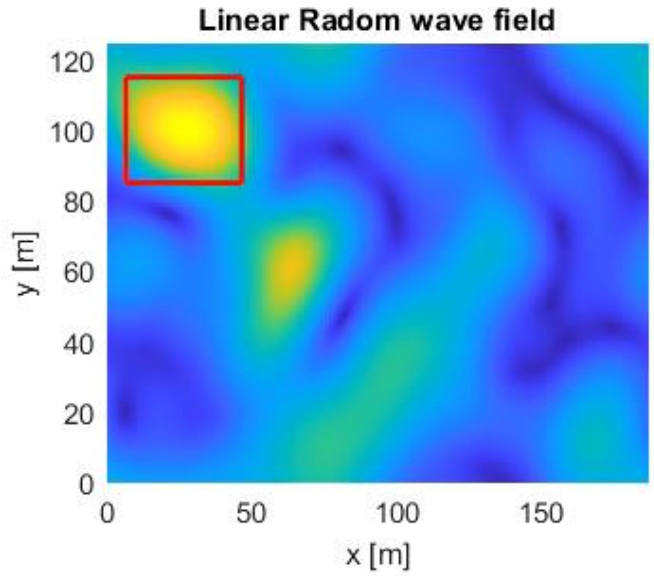
L_x: Length scale in x direction

L_y: Length scale in y direction

Parameterization of random wave fields

$$G(x,y) = \sum_{n=1}^{N} g_n(x,y),$$

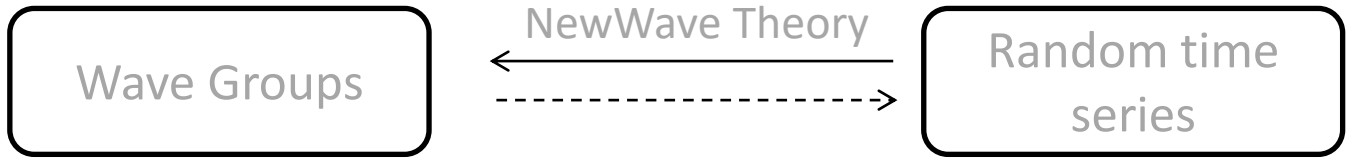




Step 1: Determine the envelope peak

Step 2: Determine the length scale parameters with optimization algorithm

A framework describing extremes



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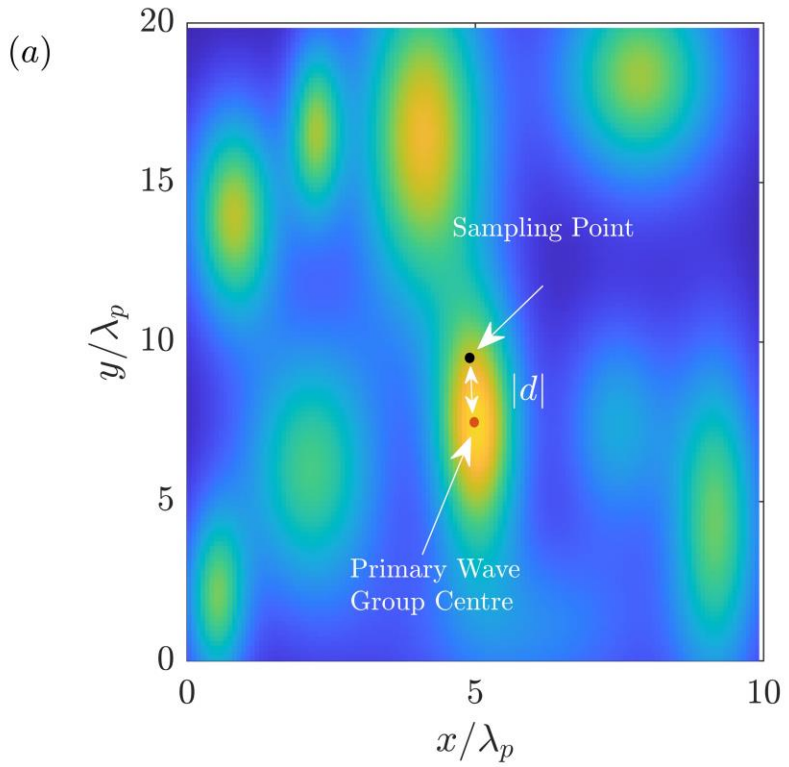
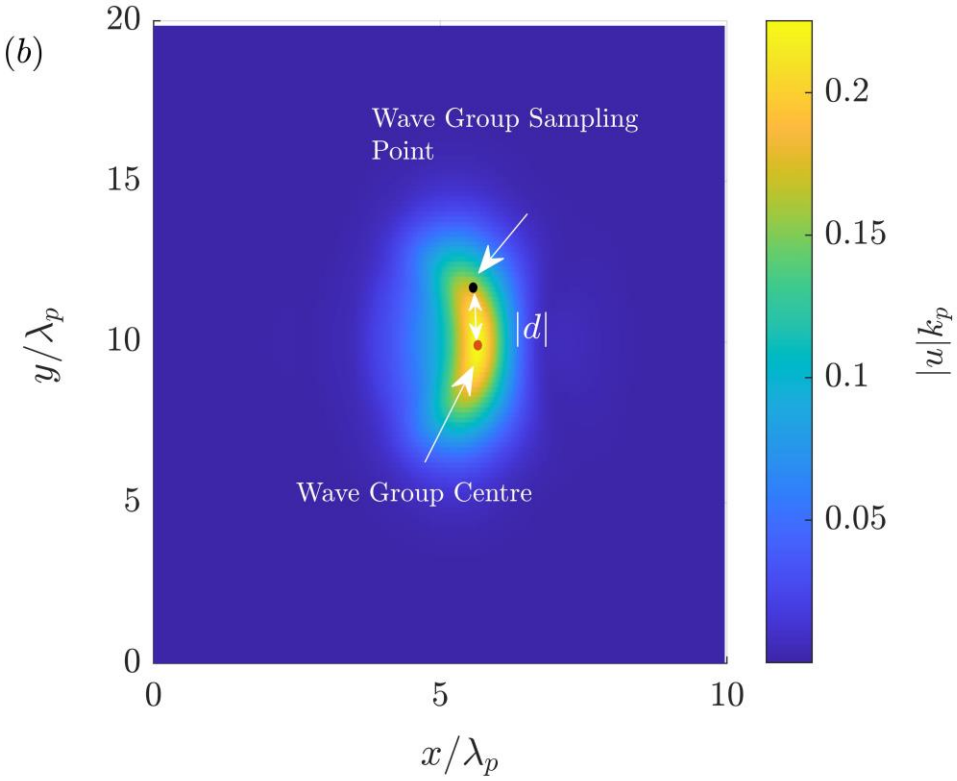
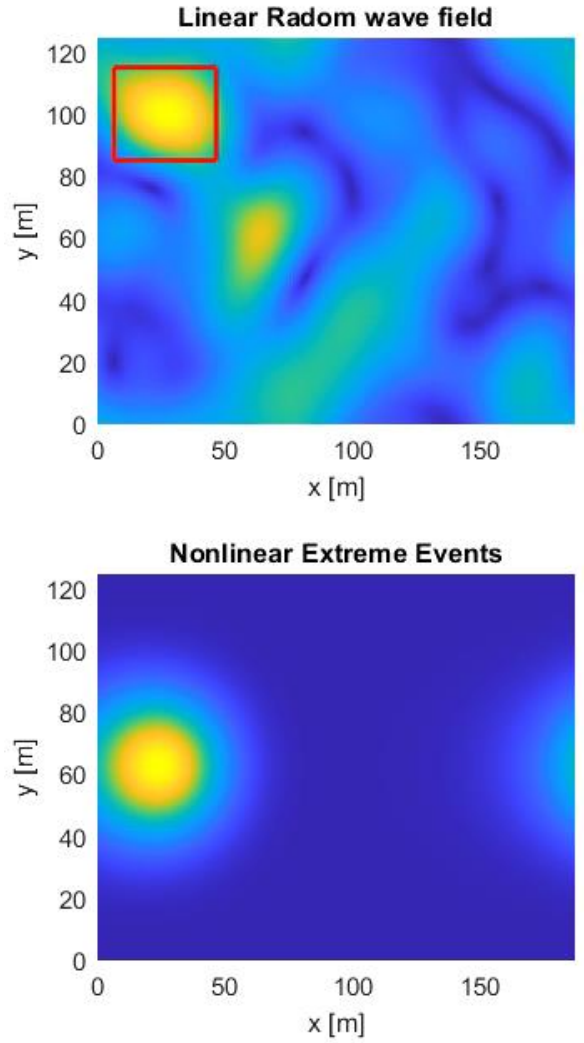
Recreate the formation of extreme events



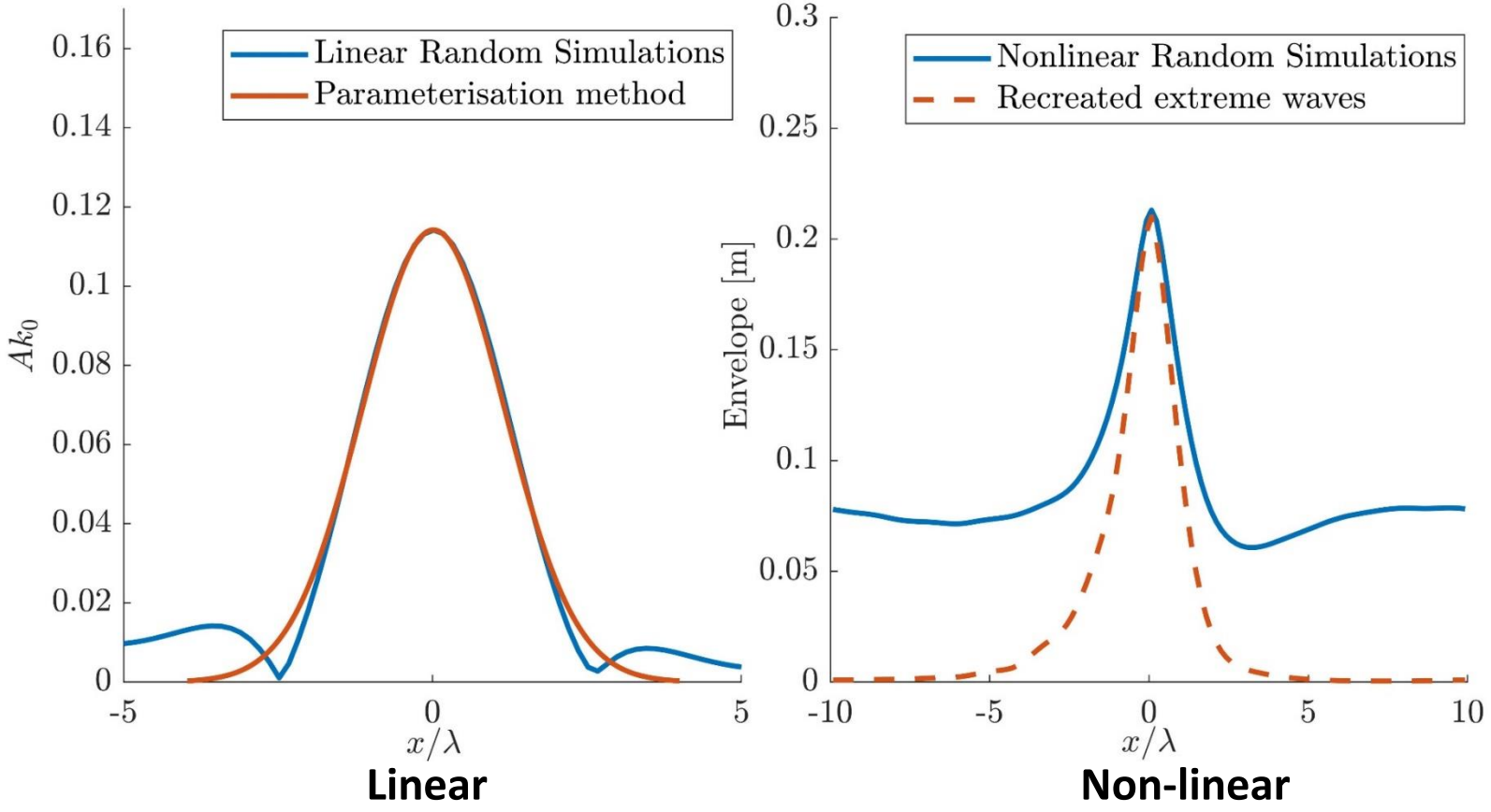
Reconstructed wave field



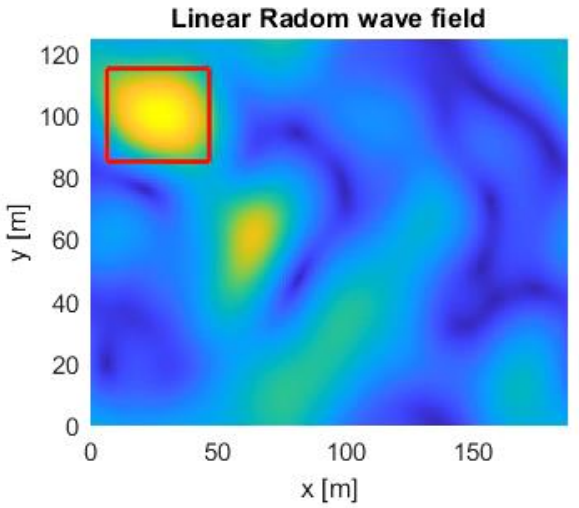
Isolated extreme events

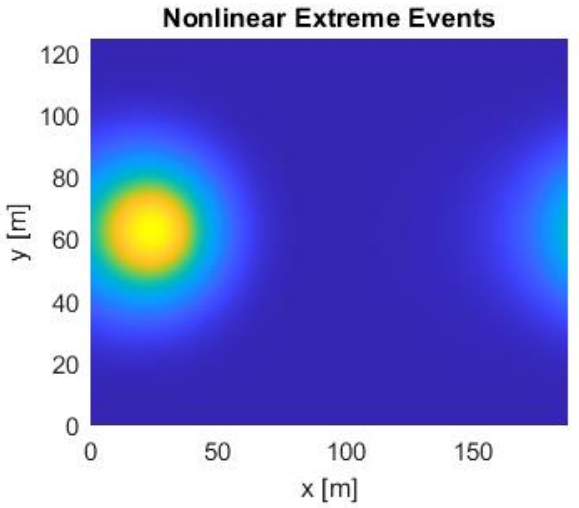


Averaged shape of extreme events



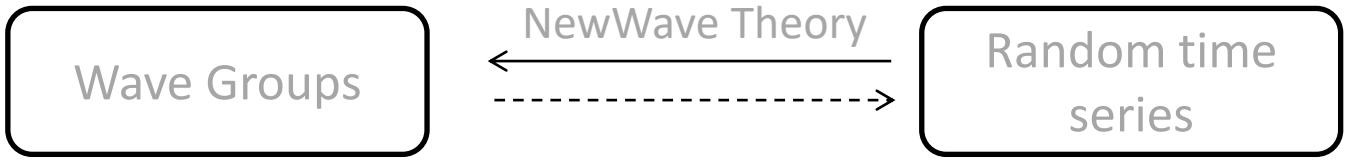
Linear **Non-linear**





- The proposed parameterization and recreation method can accurately **isolate** the extreme events and predict the **nonlinear changes**.

A framework describing extremes



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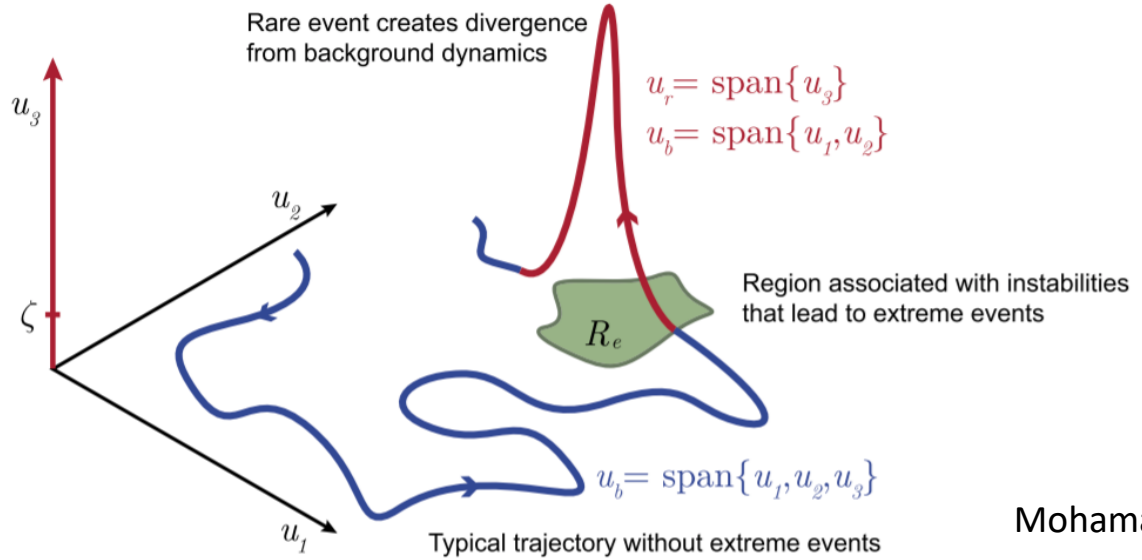
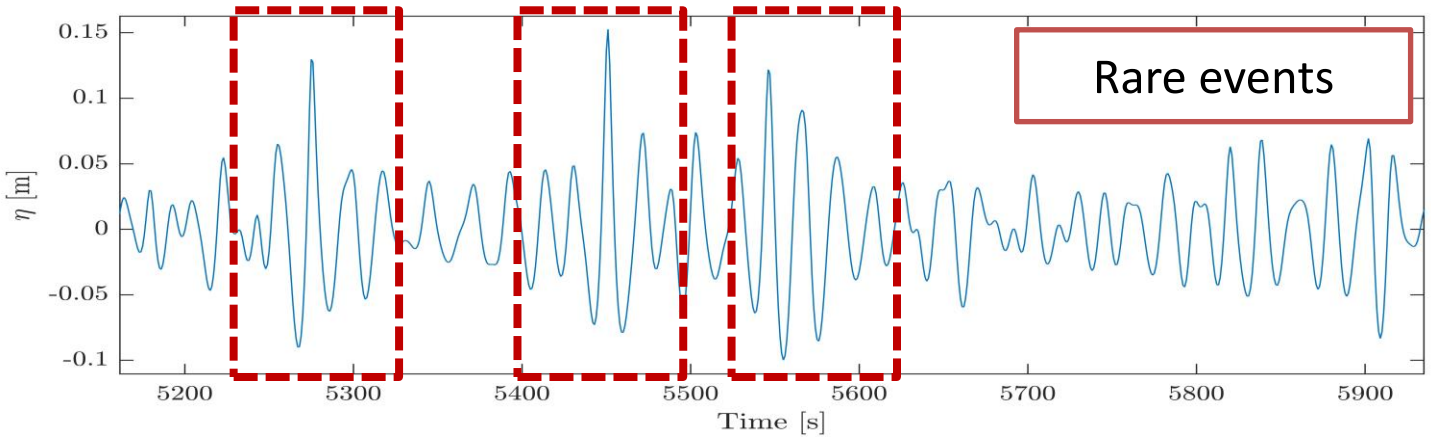
Probability of extreme events



Mohamad et. al. (2016)



Probability of extreme events

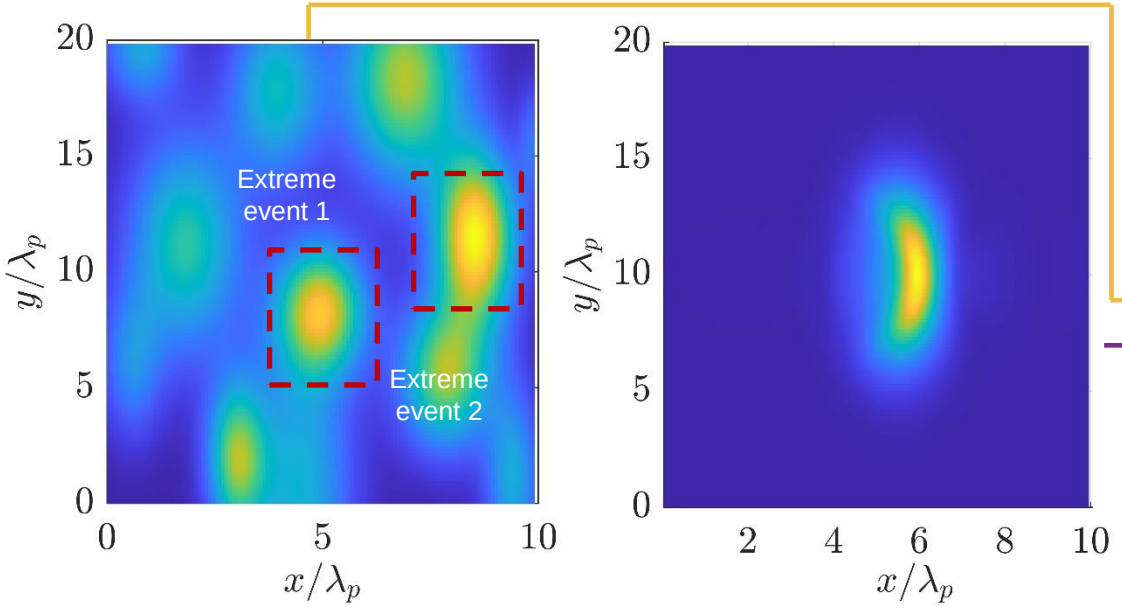
Linear theory

Probability of rare events

$$\rho(q) = \rho_l\left(q \mid R_e^c\right)\left(1 - \mathbb{P}_r\right) + \mathbb{P}_r \iiint_{R_e} \rho_r\left(q \mid A, L_x, L_y \in R_e\right) \rho_{A,L_x,L_y}\left(A, L_x, L_y \mid R_e\right) \, \mathrm{d}A \, \mathrm{d}L_x \, \mathrm{d}L_y,$$

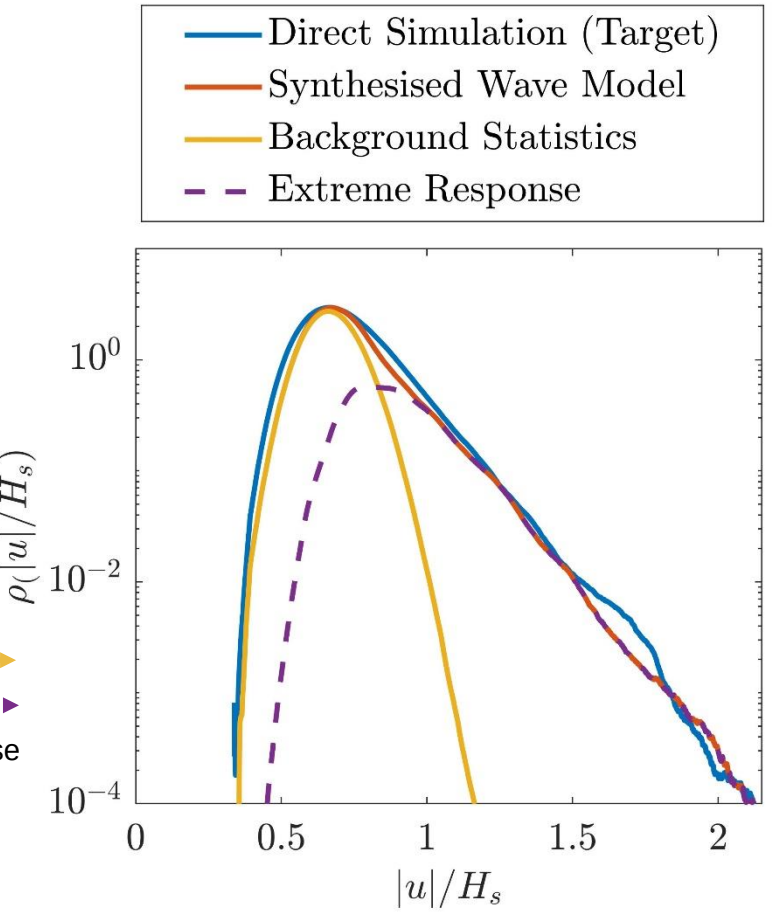
Conditional statistics of rare events

System response of rare events



Background statistics

Extreme response



Tang & Adcock (2022) Ocean Engineering
Tang & Adcock (2022) Applied Ocean Research

Conclusions

- A data-driven method is presented for parameterizing the random wave fields with deterministic wave groups.

- The nonlinear simulation of individual wave groups can capture the nonlinear changes of in the averaged shape of largest events.

- The probabilistic decomposition method can provide accurate estimation for space-time wave statistics for test cases without random time series simulations.


Thank you!