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A Bayesian hierarchical multifidelity model for turbulent flow problems

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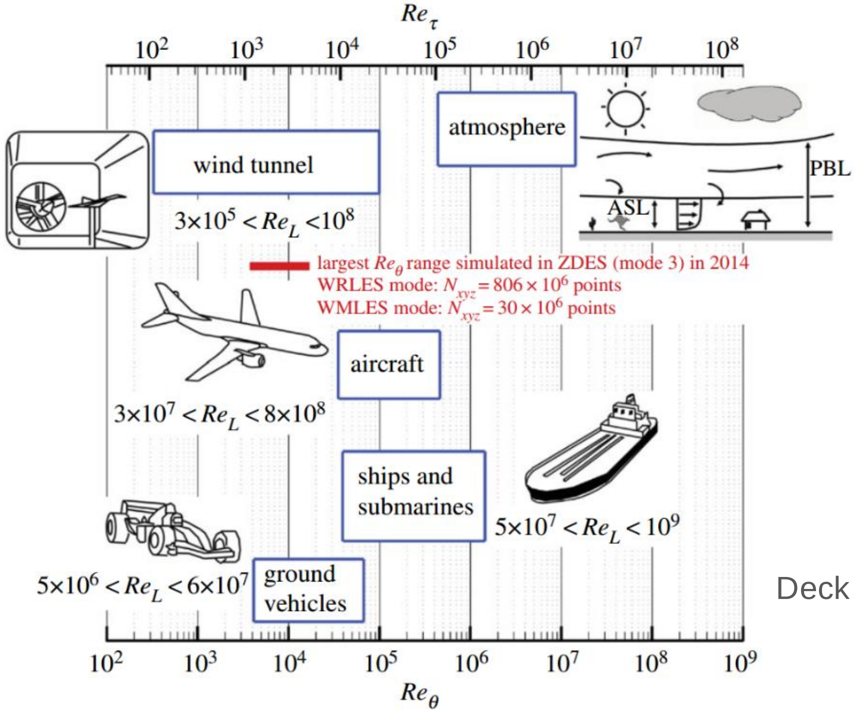
Introduction

Wall-bounded turbulent flows

Source: https://www.shutterstock.com/

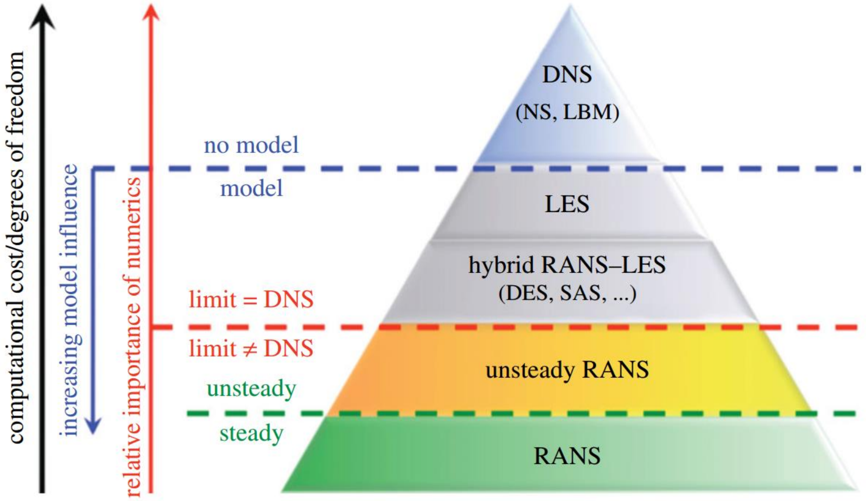


Deck et al. 2014

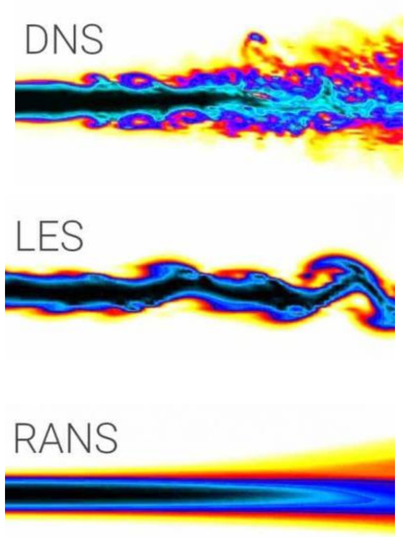
Hierarchy of Turbulence Simulation Approaches

Challenges:

- Computational cost
- Uncertainties



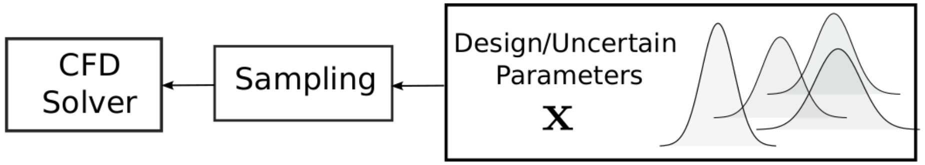
Sagaut et al. 2013

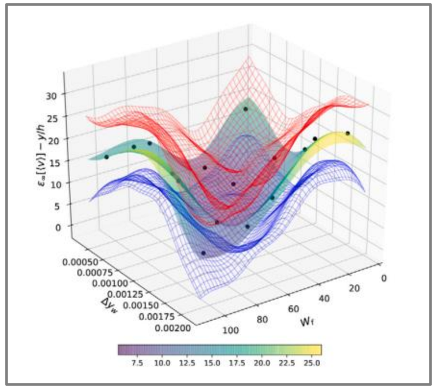


Source: https://www.idealsimulations.com/resources/turbulence-models-in-cfd/

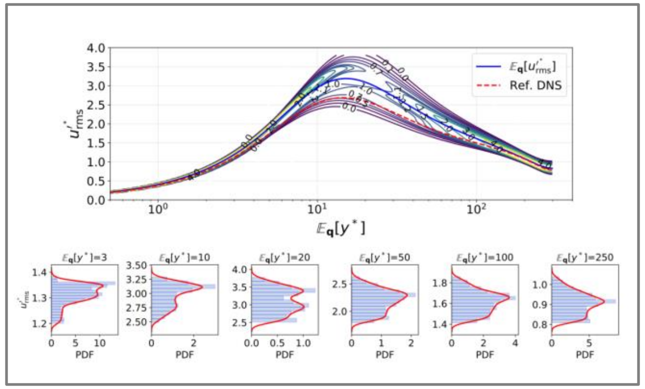
Multifidelity Models

Outer-loop Problems

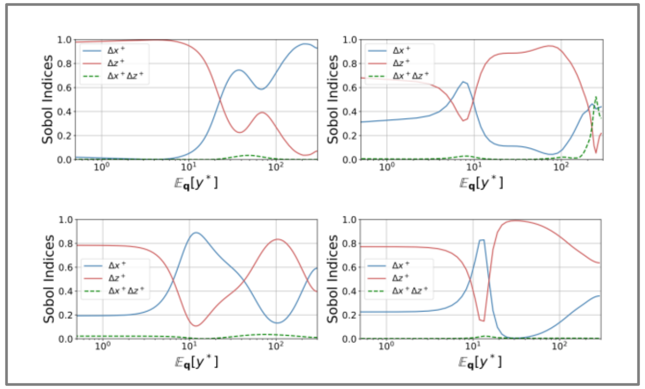




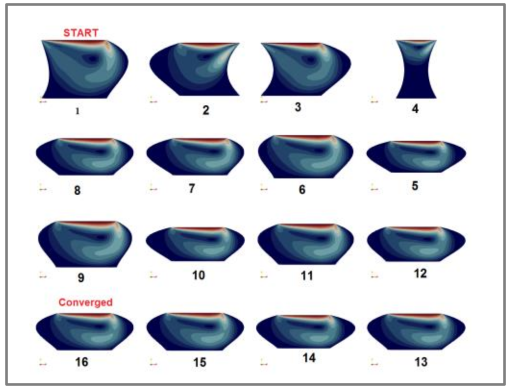
Surrogates



Uncertainty Propagation

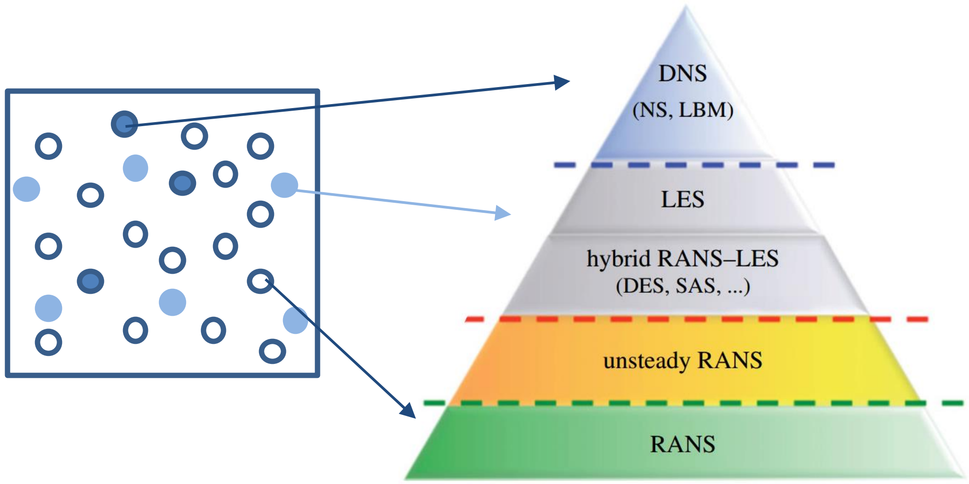


Sensitivity Analysis



Bayesian Optimization

- Several simulations of a turbulent flow are required.
- **Multifidelity Models (MFM)**: achieve high accuracy with a given limited computational budget.
- We need a MFM that:
  - is consistent with turbulence modeling hierarchy
  - can handle uncertainties.

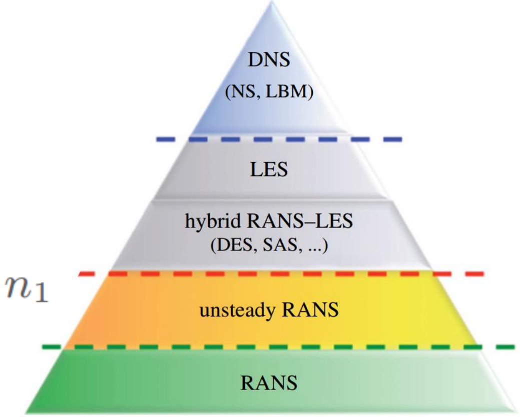


HC-MFM: Hierarchical MFM with Calibration

Goh et al. Technometrics, 55(4):501–512, 2013

HC-MFM

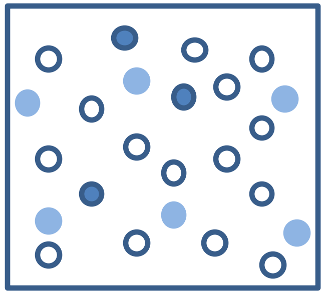
$$
\begin{cases}
y_{M_1}(\mathbf{q}_i) = \hat{f}(\mathbf{q}_i, \boldsymbol{\theta}_3, \boldsymbol{\theta}_s) + \hat{g}(\mathbf{q}_i, \boldsymbol{\theta}_2, \boldsymbol{\theta}_s) + \hat{\delta}(\mathbf{q}_i) + \varepsilon_{1_i} & , \quad i = 1, 2, \cdots, n_1 \\
y_{M_2}(\mathbf{q}_i) = \hat{f}(\mathbf{q}_i, \boldsymbol{\theta}_3, \mathbf{t}_{s_i}) + \hat{g}(\mathbf{q}_i, \mathbf{t}_{2_i}, \mathbf{t}_{s_i}) + \varepsilon_{2_i} & , \quad i = 1 + n_1, \cdots, n_2 + n_1 \\
y_{M_3}(\mathbf{q}_i) = \hat{f}(\mathbf{q}_i, \mathbf{t}_{3_i}, \mathbf{t}_{s_i}) + \varepsilon_{3_i} & , \quad i = 1 + n_2 + n_1, \cdots, n_3 + n_2 + n_1
\end{cases}
$$



Global Kernel Matrix

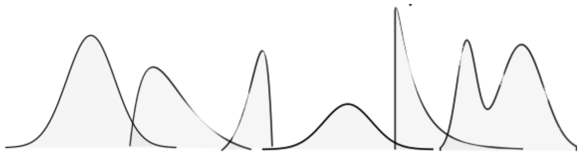
$$
\boldsymbol{\Sigma} = \boldsymbol{\Sigma}_f + \begin{bmatrix} \boldsymbol{\Sigma}_g & \mathbf{0}_{(n_1+n_2)\times n_3} \\ \mathbf{0}_{n_3\times(n_1+n_2)} & \mathbf{0}_{n_3\times n_3} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Sigma}_\delta & \mathbf{0}_{n_1\times(n_2+n_3)} \\ \mathbf{0}_{(n2+n3)\times n_1} & \mathbf{0}_{(n_2+n_3)\times(n_2+n_3)} \end{bmatrix}
$$

$$
+ \begin{bmatrix} \boldsymbol{\Sigma}_{\varepsilon_1} & \mathbf{0}_{n_1\times n_2} & \mathbf{0}_{n_1\times n_3} \\ \mathbf{0}_{n_2\times n_1} & \boldsymbol{\Sigma}_{\varepsilon_2} & \mathbf{0}_{n_2\times n_3} \\ \mathbf{0}_{n_3\times n_1} & \mathbf{0}_{n_3\times n_2} & \boldsymbol{\Sigma}_{\varepsilon_3} \end{bmatrix}
$$



Data samples from various fidelities

Bayesian Inference using MCMC methods



S. R., T. Mukha, P. Schlatter, arXiv:2210.14790, 2022.

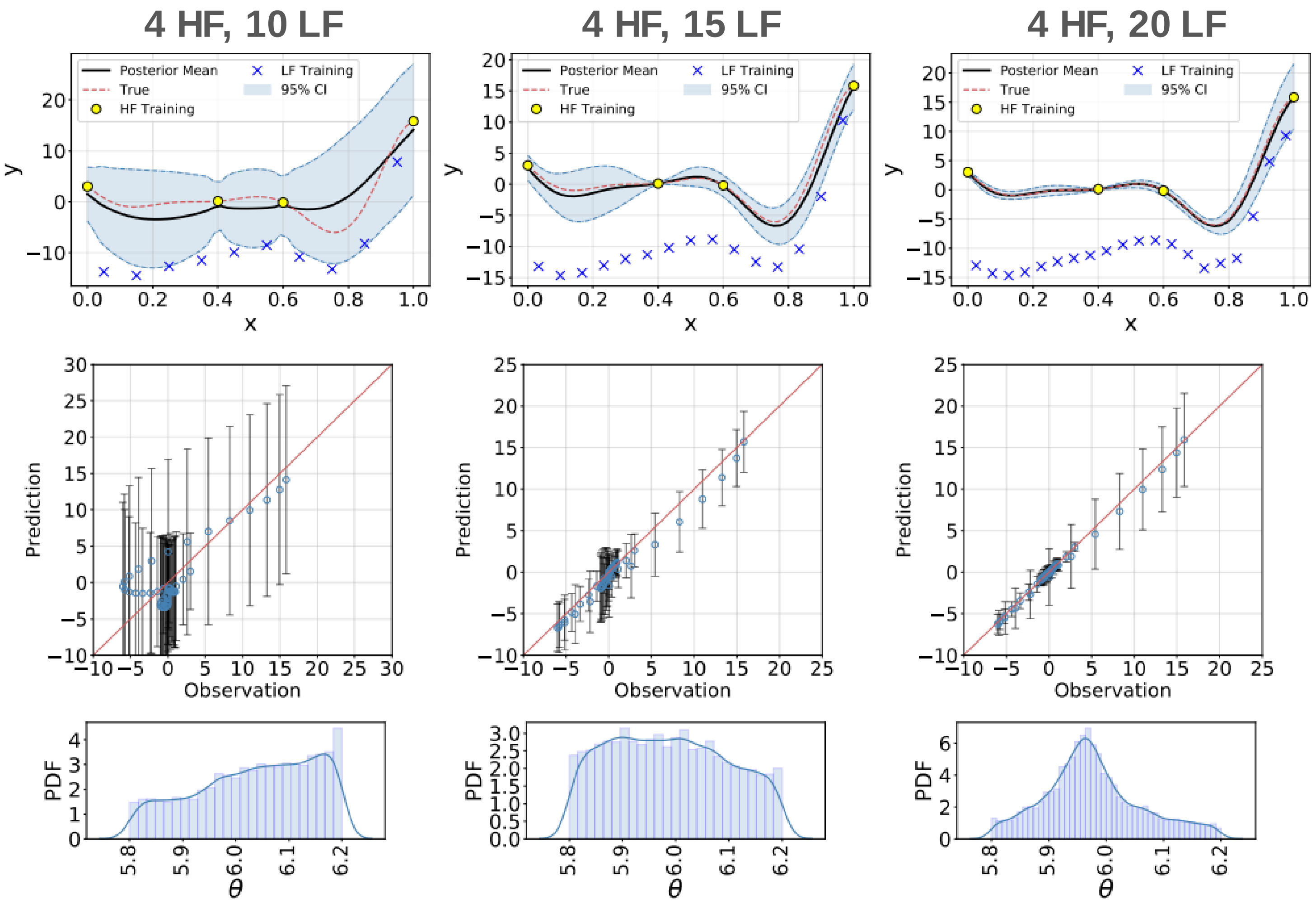
Application to a Toy Problem

Forrester et al. 2007: $\begin{cases} y_H(x) = (\theta x - 2)^2 \sin(2\theta x - 4) & , x \in [0,1] \\ y_L(x) = y_H(x) + 10(x - 0.5) + 10 \end{cases}$

- Prior distribution for θ: $t \sim \mathcal{U}[5.8, 6.2]$
- True θ is 6

Fix HF data & increase LF data:

- More accurate predictions,
- More informative posterior for parameter.

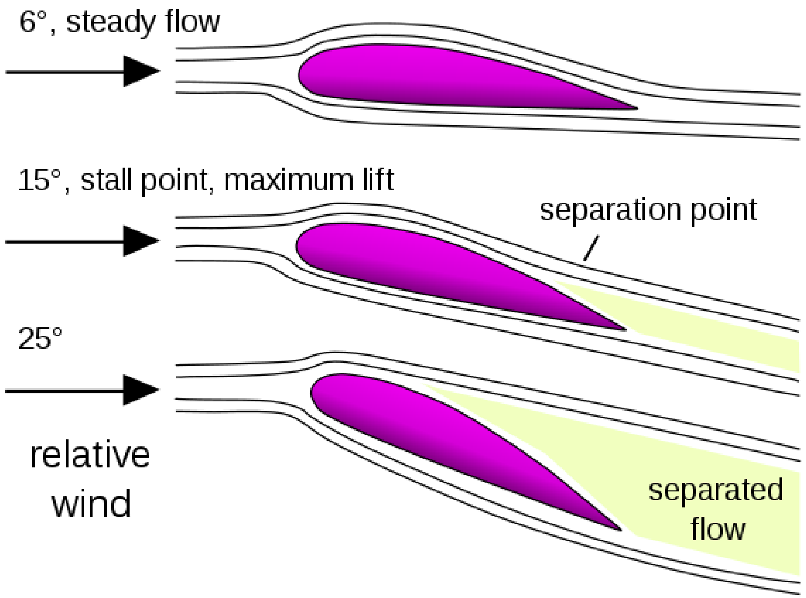


Polars for an Airfoil

- NACA0015 airfoil at $\mathrm{Re} = 1.6 \times 10^6$
  - **Input: AoA** (angle of attack)
  - **Output: CL, CD** (lift & drag coefficients)

Model Hierarchy:

- M_1: **Wind tunnel experiments**
  - Bertagnolio 2008, LM Glasfiber wind tunnel
  - Inlet turbulence intensity (T.I.): 0.1%
  - AoA $\in [0^\circ, 19^\circ]$
- M_2: **Detached Eddy Simulation (DES)**
  - DES: A hybrid RANS/LES approach
  - Data: Gilling, Sørensen & Davidson 2009
  - AoA $\in [8^\circ, 19^\circ]$
  - $T.I. \in [0\%, 2\%]$
- M_3: **2D Reynolds-averaged Navier-Stokes (RANS) simulations**
  - Data: Gilling, Sørensen & Davidson 2009
  - AoA $\in [0^\circ, 20^\circ]$



https://en.wikipedia.org/wiki/Stall_(fluid_dynamics)

F. Bertagnolio. NACA0015 Measurements in LM Wind Tunnel and Turbulence Generated Noise, 2008 [2].
L. Gilling, N. Sorensen, and L. Davidson, AIAA (24)2009-270, 2009 [6].

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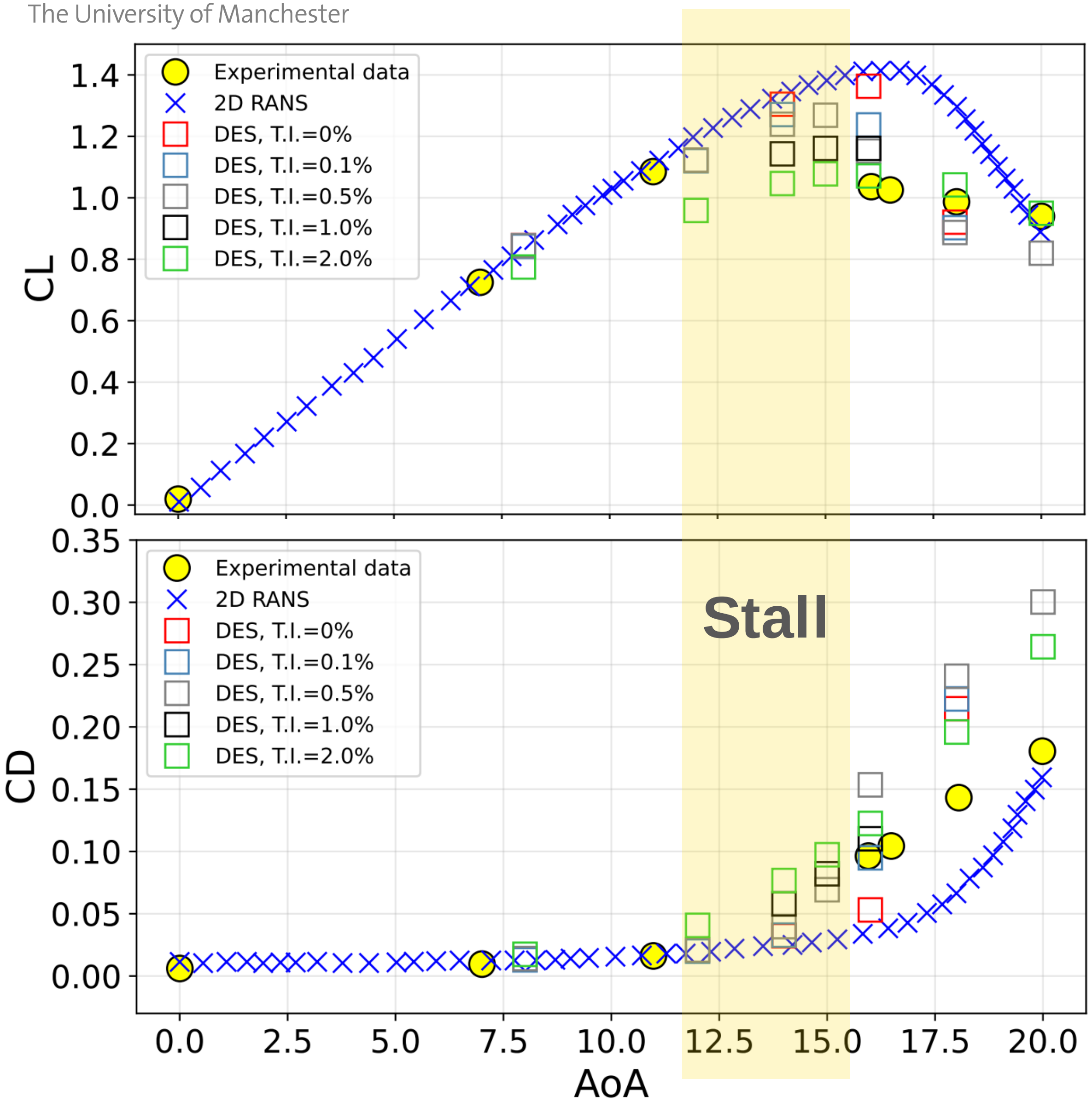
Polars for an Airfoil: challenge

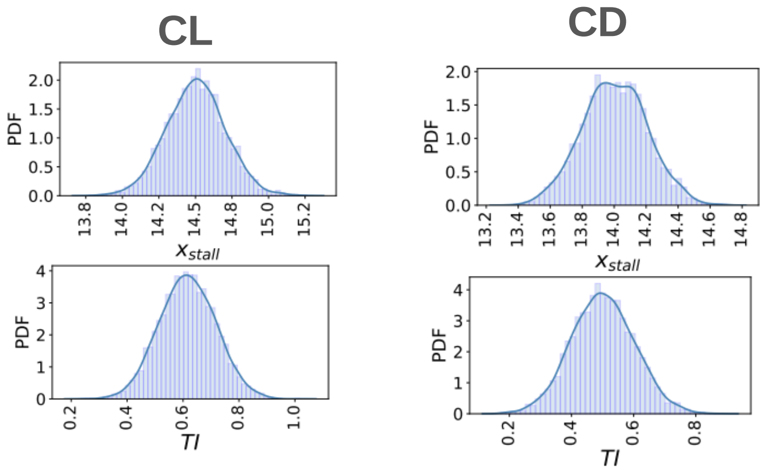


The HC-MFM should capture the stall =>
Kernel of the model discrepancy is modified.

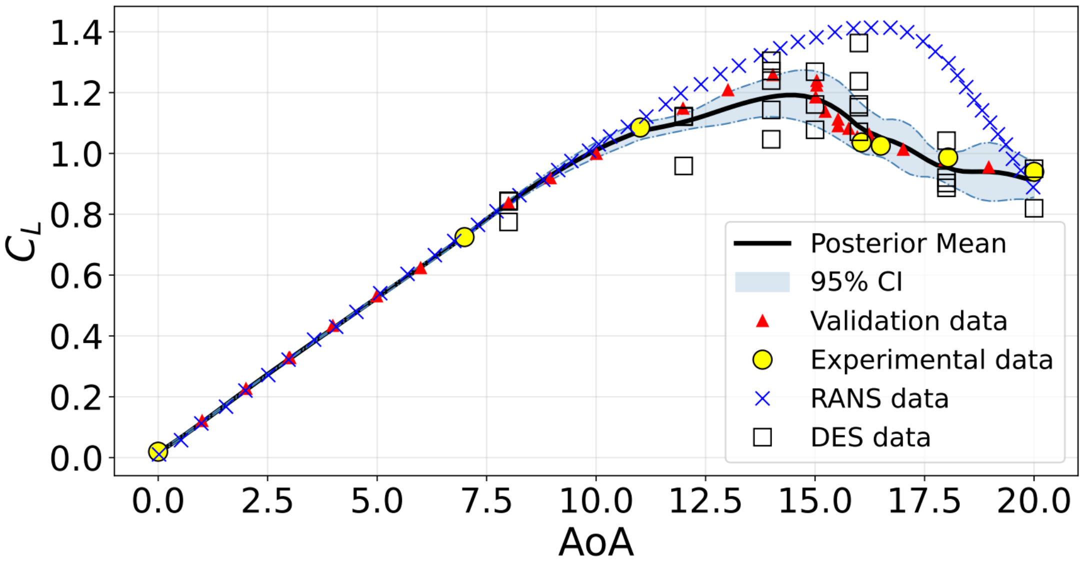
$$y_{M_1}(\mathbf{x}_i) = \hat{f}(\mathbf{x}_i, \boldsymbol{\theta}_3, \boldsymbol{\theta}_s) + \hat{g}(\mathbf{x}_i, \boldsymbol{\theta}_2, \boldsymbol{\theta}_s) + \hat{\delta}(\mathbf{x}_i) + \varepsilon_{1_i}$$

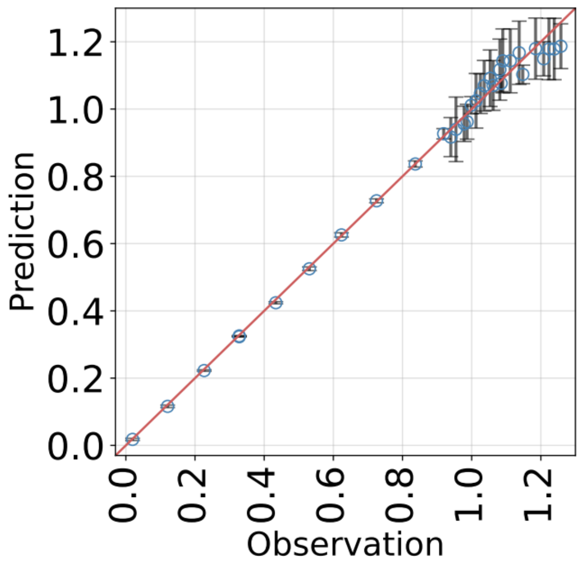
$$\Sigma_{\delta_{ij}} = \lambda^2_{\delta_1} \varphi(x_i) k_{\delta_1}(\bar{d}_{\delta_{ij}}) \varphi(x_j) + \lambda^2_{\delta_2} \varphi(x_i) k_{\delta_2}(\bar{d}_{\delta_{ij}}) \varphi(x_j)$$

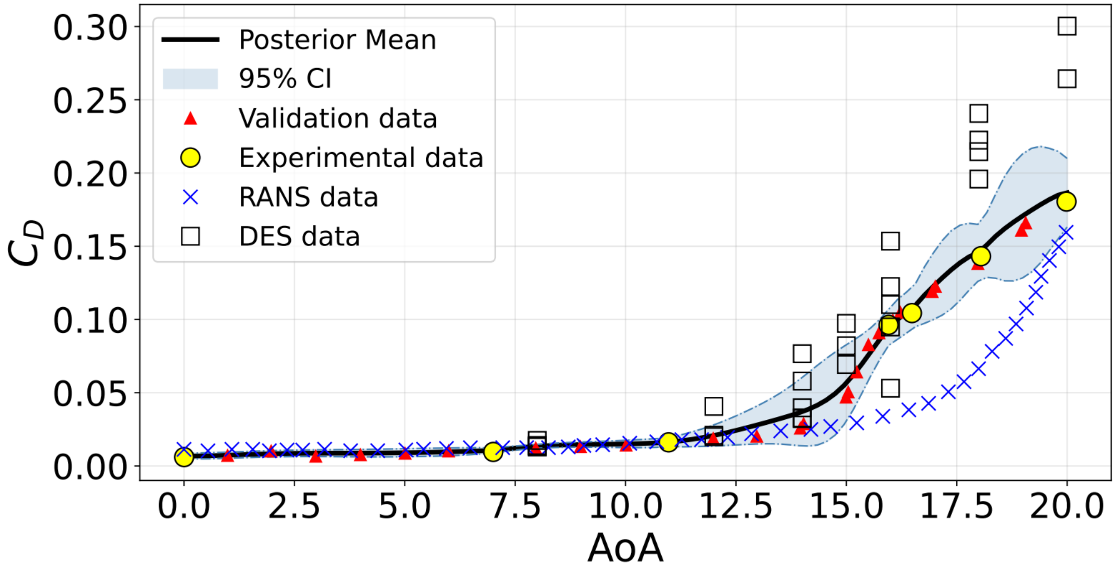
$$\varphi(x) = [1 + \exp(-\alpha_{\text{stall}}(x - x_{\text{stall}}))]^{-1}$$

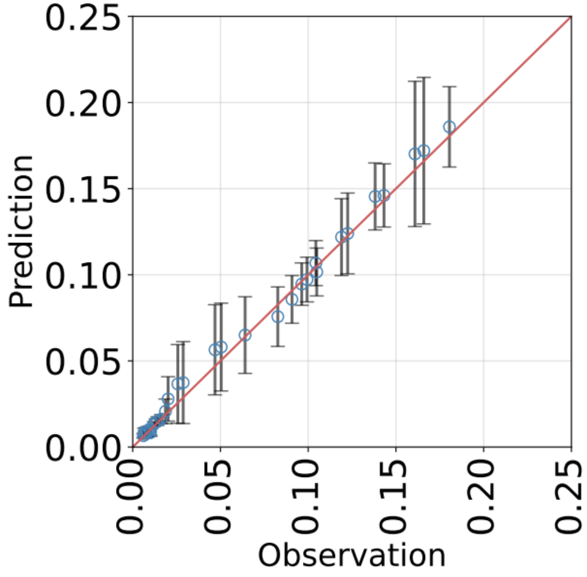


Polars for an Airfoil



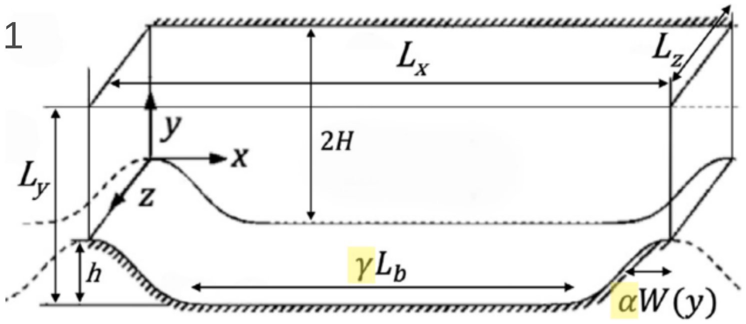


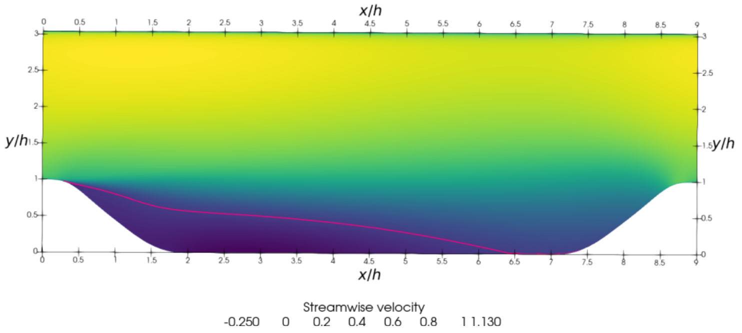




Geometrical Uncertainties in Periodic-hill Flow

By Voet et al., 2021

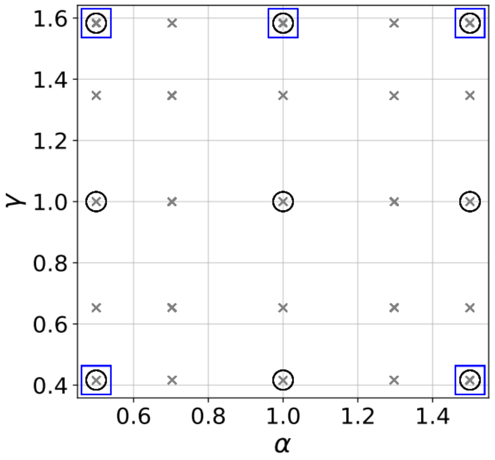


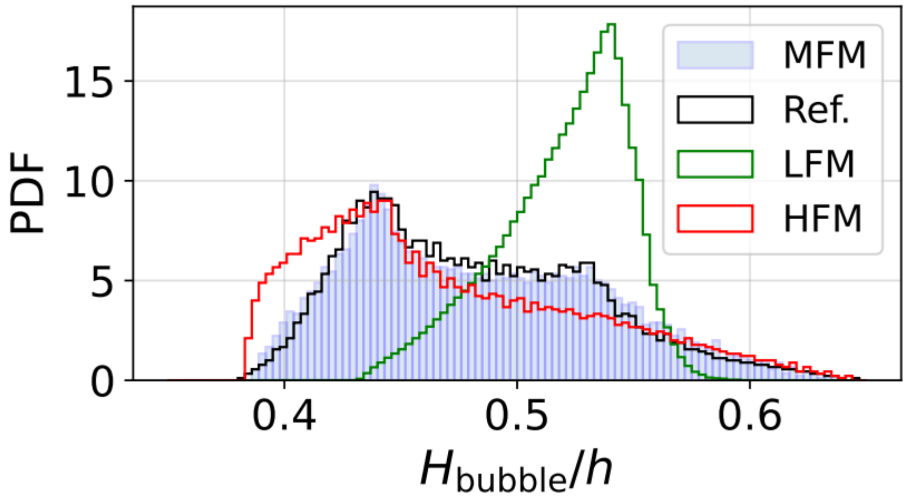


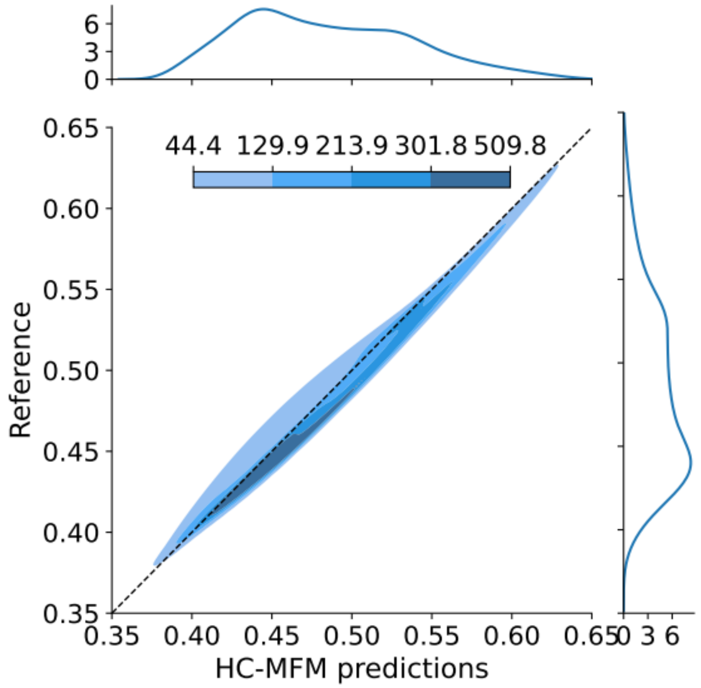
- $Re = 5600$
- For $\alpha \sim \mathcal{U}[0.45, 1.65]$ and $\gamma \sim \mathcal{U}[0.375, 1.74]$, estimate the PDF of the QoIs.
- QoI: Height of the circulation bubble at some x/h
- Studied by Voet et al. 2021 using PCE-based and co-Kriging MFMs.
- M_1: **DNS**
  - Xiao et al. 2020
- M_2: **RANS**
  - ANSYS Fluent 2019R3, $k - \omega$ SST model with default coefficients except κ with prior $\kappa \sim \mathcal{U}[0.3, 0.5]$
  - Total 125 RANS simulations: $5 \times 5 \times 5$ Gauss-Legendre samples for $\alpha \times \gamma \times \kappa$
  - Computional mesh: $150 - 500 \times 10^3$ cells

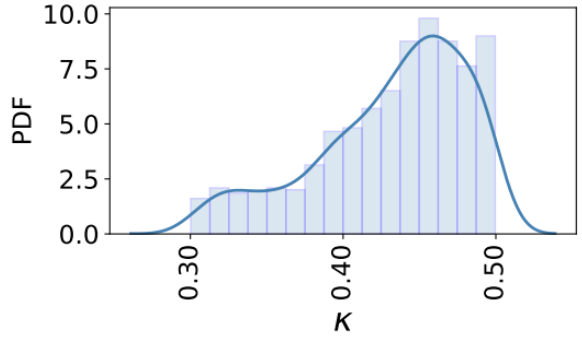
Geometrical Uncertainties in Periodic-hill Flow Predictions by the HC-MFM

HF: 5 DNS
LF: 125 RANS
Ref.: 9 DNS*









Posterior of k, RANS modeling parameter

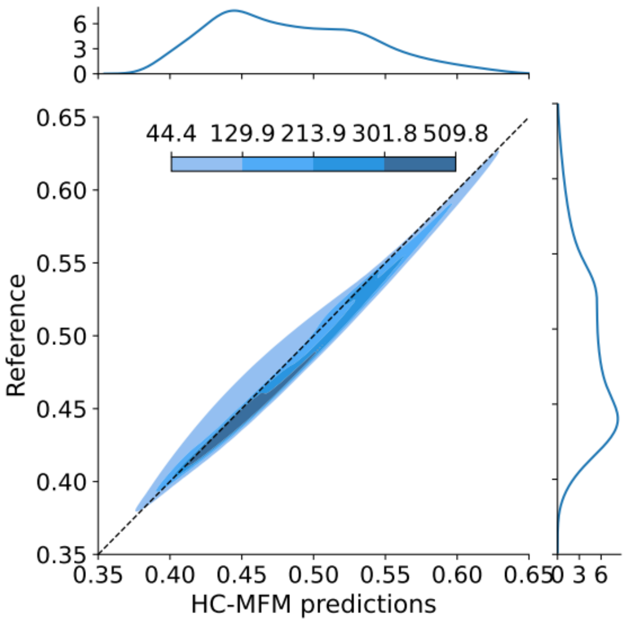
*DNS data: H. Xiao et al., Computers & Fluids, 200:104431, 2020

Geometrical Uncertainties in Periodic-hill Flow
Impact of the inference method

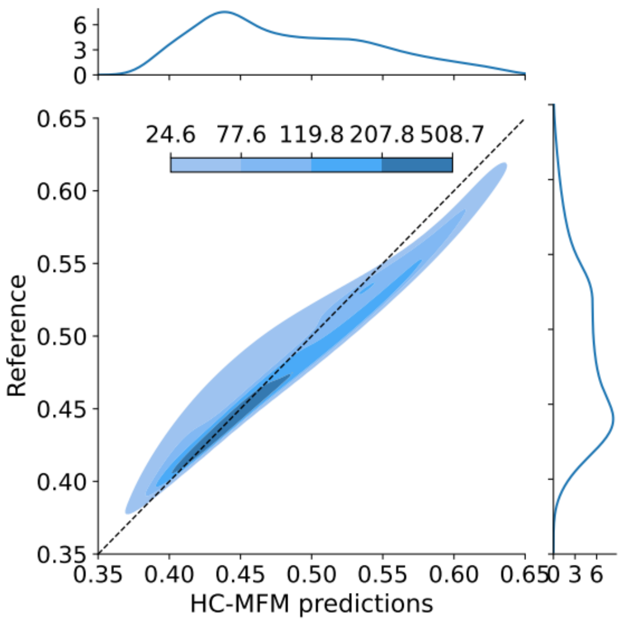
Bayes' formula for inverse problems

$$p(\boldsymbol{\theta}, \boldsymbol{\beta}|\mathcal{D}) \propto p(\mathcal{D}|\boldsymbol{\theta}, \boldsymbol{\beta})\, p(\boldsymbol{\theta})\, p(\boldsymbol{\beta})$$

MCMC sampling
(Markov Chain Monte Carlo)



MAP Estimator
(Maximum a-posteriori)



Conclusions

https://doi.org/10.48550/arXiv.2210.14790

- Promising results by adapting the HC-MFM of Goh et al. 2013 to turbulent flow applications.
- HC-MFM is generative and can account for modeling and observational uncertainties.
- For fixed HF data, HC-MFM priotorizes the accuracy of the predictions over the calibration of fidelity-related parameters.
- Keeping HF data fixed and increasing the number of LF data improves the accuracy of the predictions and lead to more informative posteriors for parameters.
- Based on our analyses, adopting an MCMC method for Bayesian inference is essential as point estimators lead to inaccurate results.
- HC-MFM can be applied to other applications.

Thank you!

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