Hard-constrained Thermoacoustic Neural Networks

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Imperial College London

European Research Council
Established by the European Commission

The Alan Turing Institute

Outline

- Introduction
- Background
  - Physical model
  - Data-driven model
- Thermoacoustic neural networks
- Results
- Conclusion

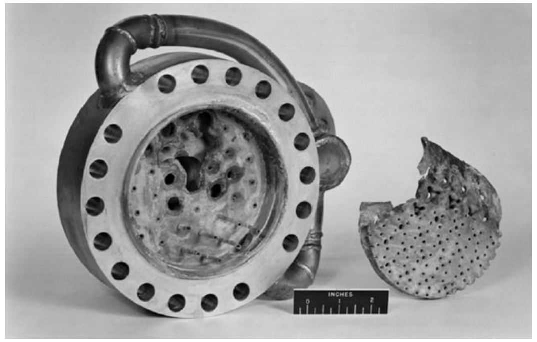
Thermoacoustics and low-order models

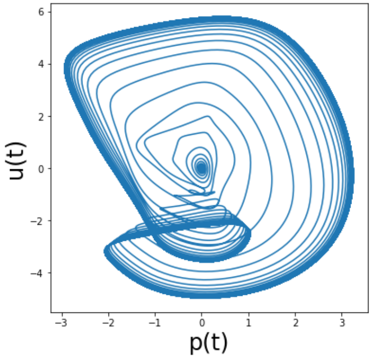
- Heat release rate sufficiently in phase with pressure ⇒ **thermoacoustic oscillations**

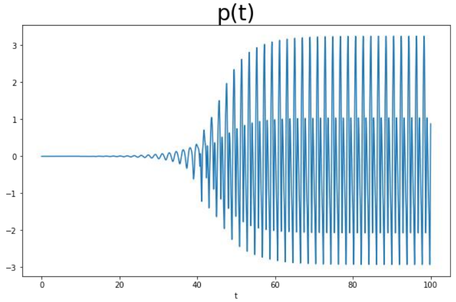
Typical regimes are

  - **limit-cycles** with finite amplitude
  - **quasiperiodic** and **chaotic** oscillations

- **Low-order models** are helpful for prediction, design and control







Objective

Robust, generalizable low-order model for thermoacoustic solutions

- using as **little data** as possible
- flow reconstruction from **partial state observations**

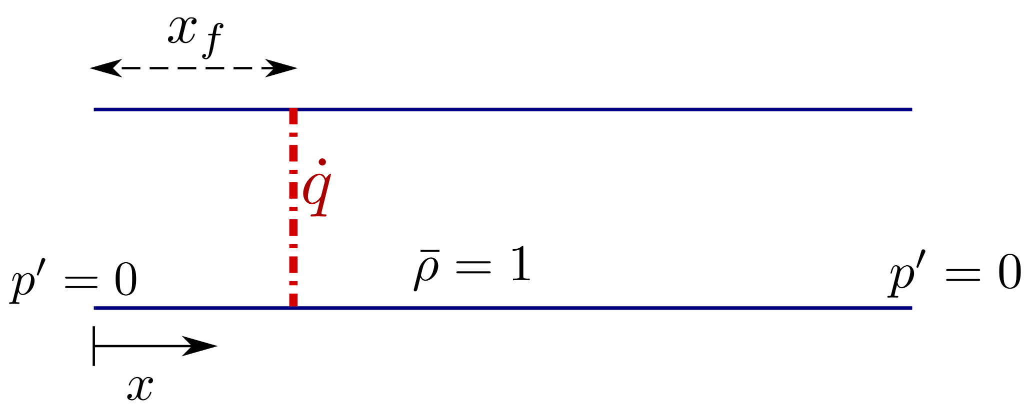
How?

Robust, generalizable low-order model for thermoacoustic solutions

Constrain properties of the solution (function) space in the neural network

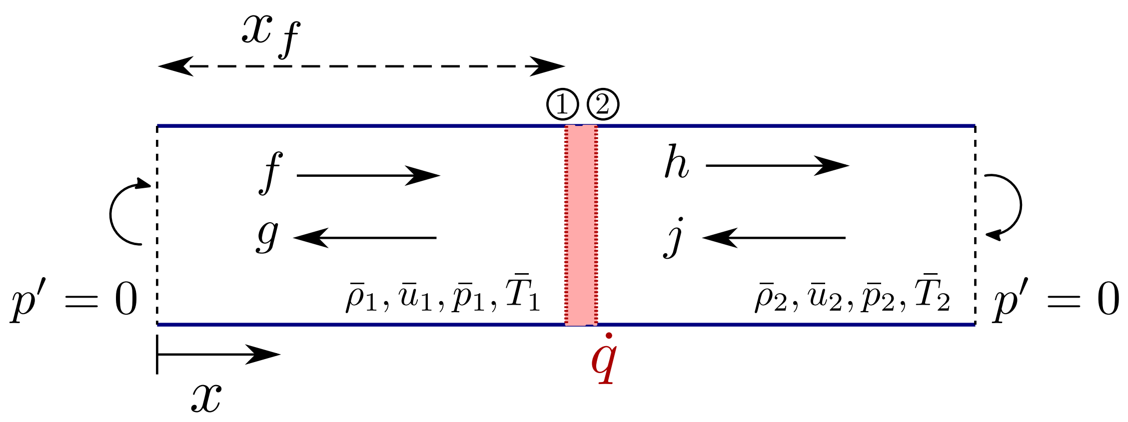
Rijke tube

Physical low-order model



- Nonlinear, time-delayed heat release law
- Solution by Galerkin decomposition

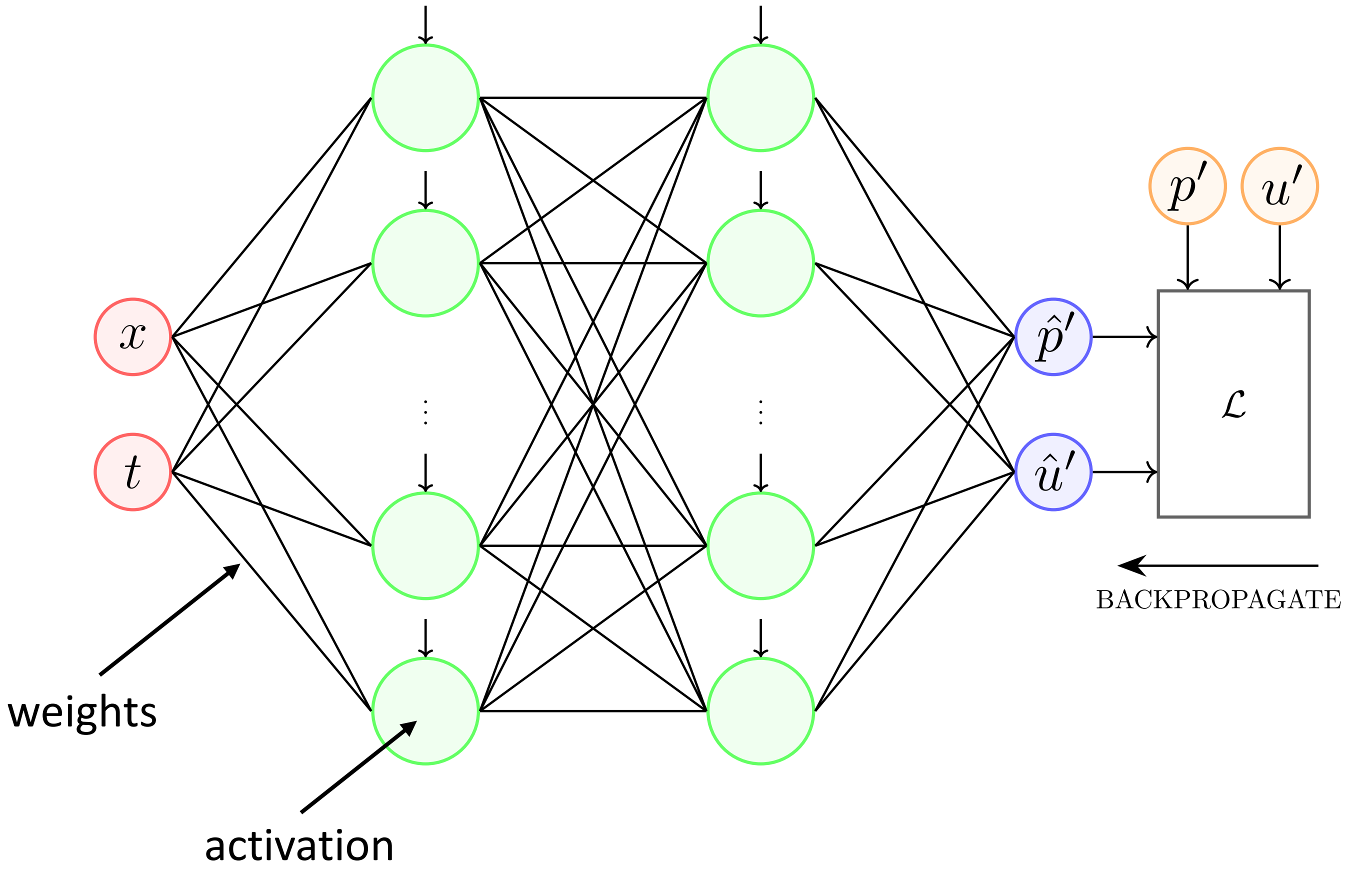
More realistic benchmark model



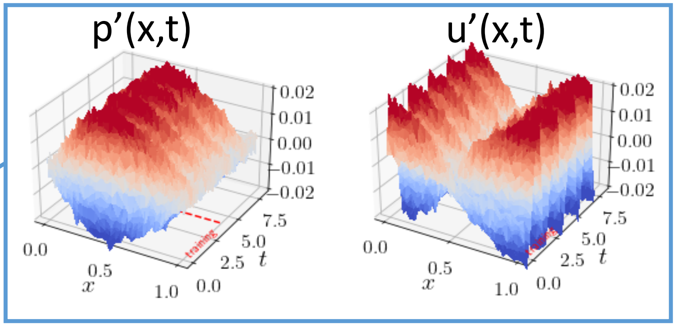
- Kinematic flame model
- Mean flow effects and temperature jump across the flame
- Solution by travelling wave approach

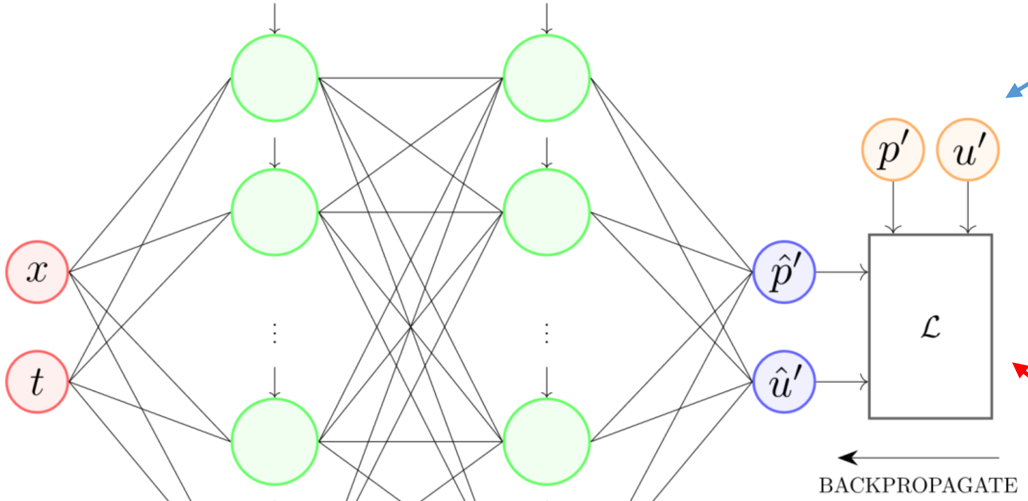
Recap: Feedforward neural networks

- a nonlinear mapping between an input and an output vector

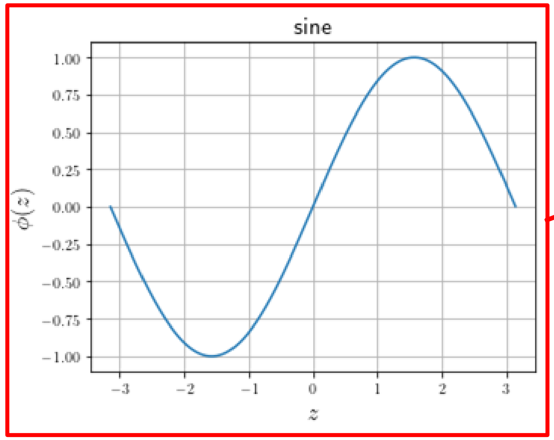


How to constrain prior knowledge?
Thermoacoustic neural networks





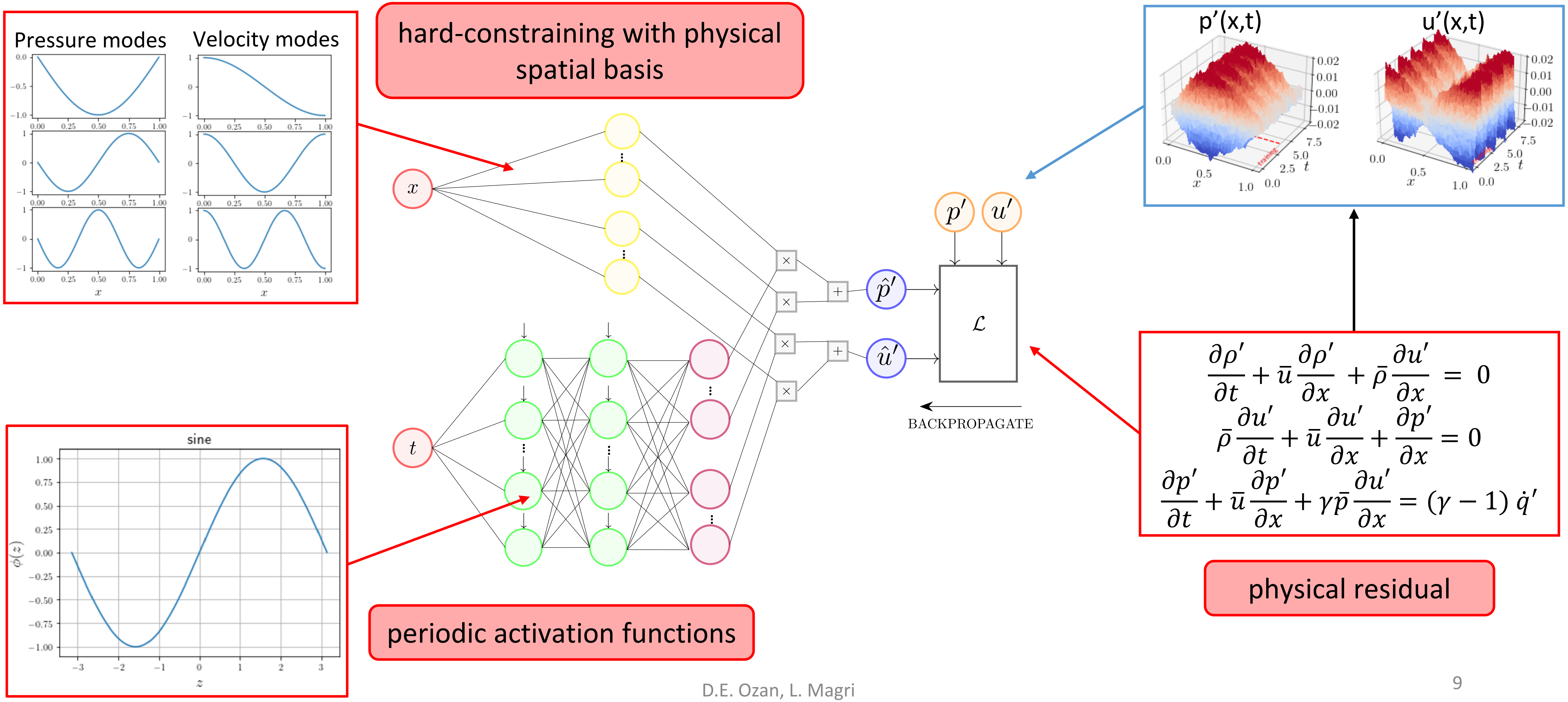
$$\frac{\partial \rho'}{\partial t} + \bar{u}\frac{\partial \rho'}{\partial x} + \bar{\rho}\frac{\partial u'}{\partial x} = 0$$
$$\bar{\rho}\frac{\partial u'}{\partial t} + \bar{u}\frac{\partial u'}{\partial x} + \frac{\partial p'}{\partial x} = 0$$
$$\frac{\partial p'}{\partial t} + \bar{u}\frac{\partial p'}{\partial x} + \gamma\bar{p}\frac{\partial u'}{\partial x} = (\gamma - 1)\,\dot{q}'$$



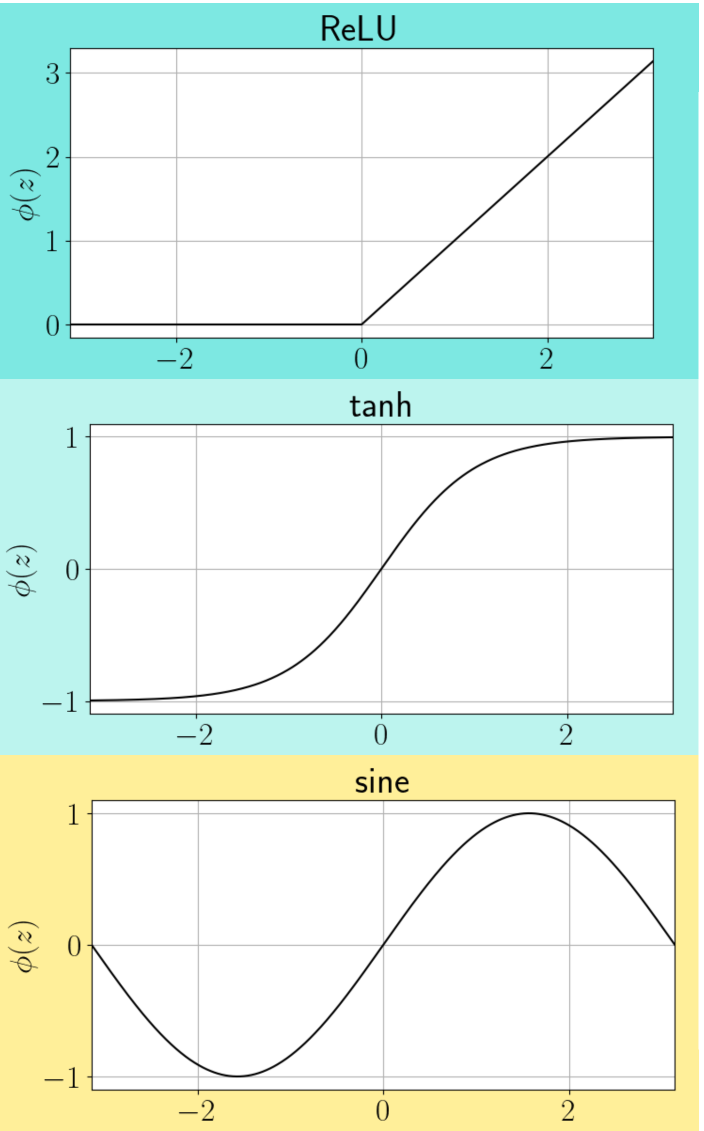
periodic activation functions

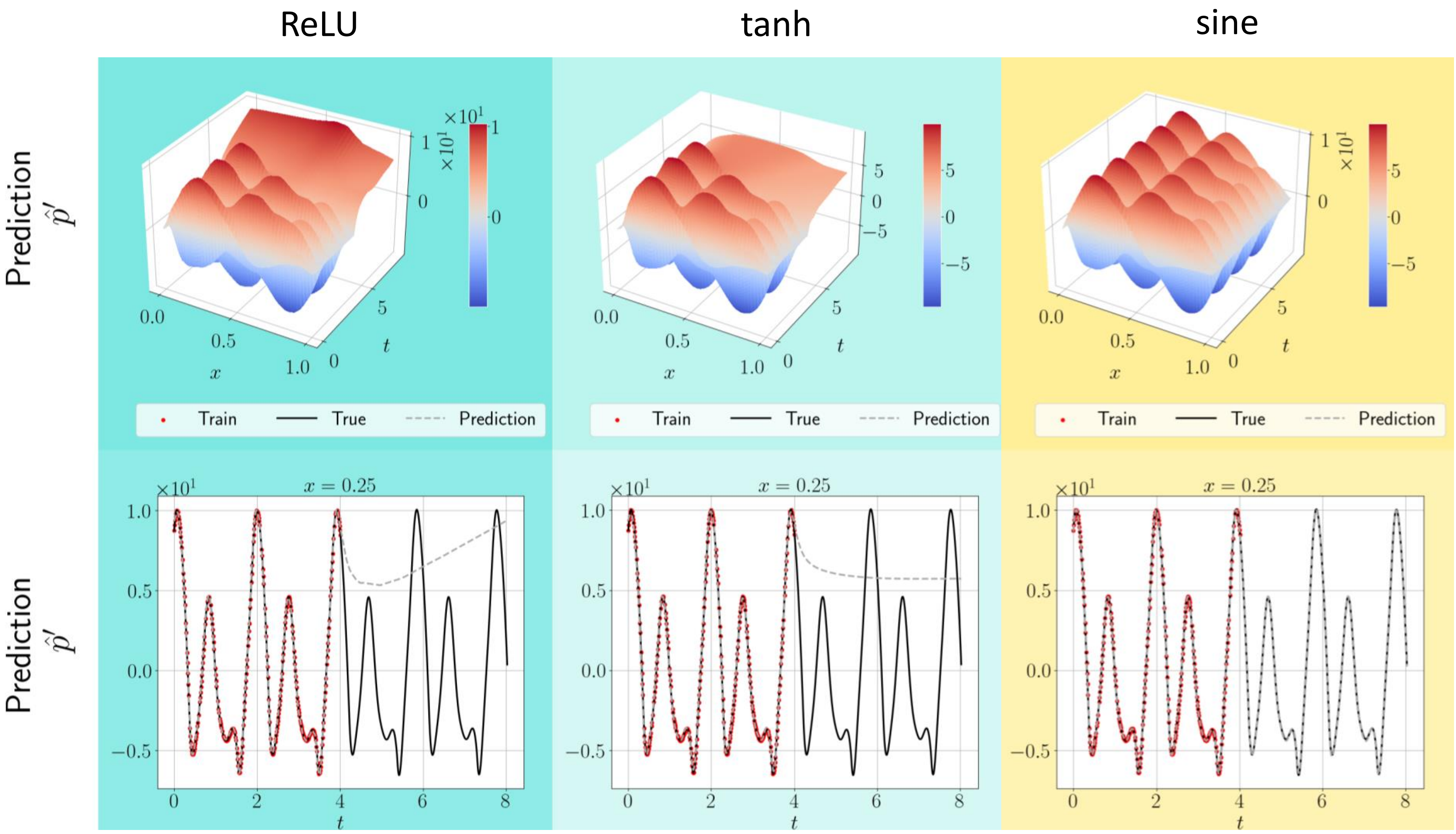
physical residual

How to constrain prior knowledge?
Thermoacoustic neural networks

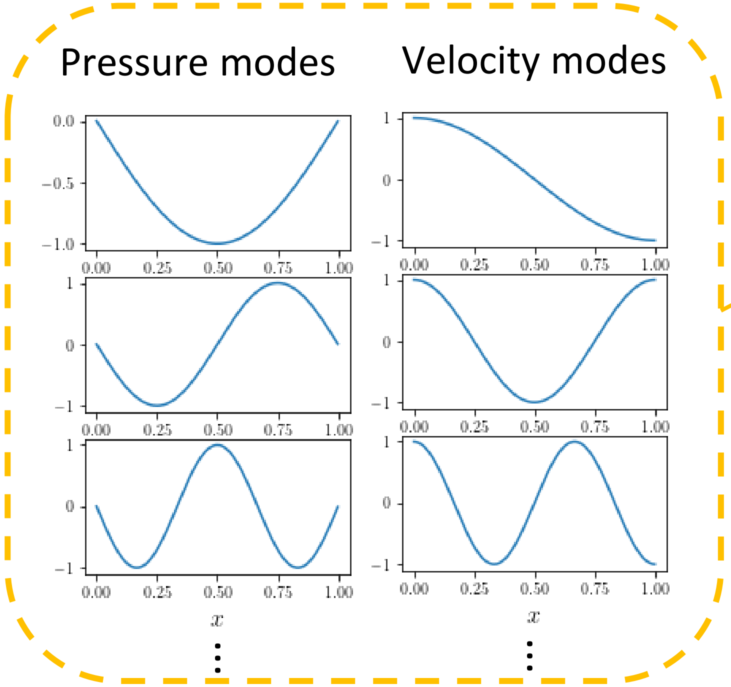


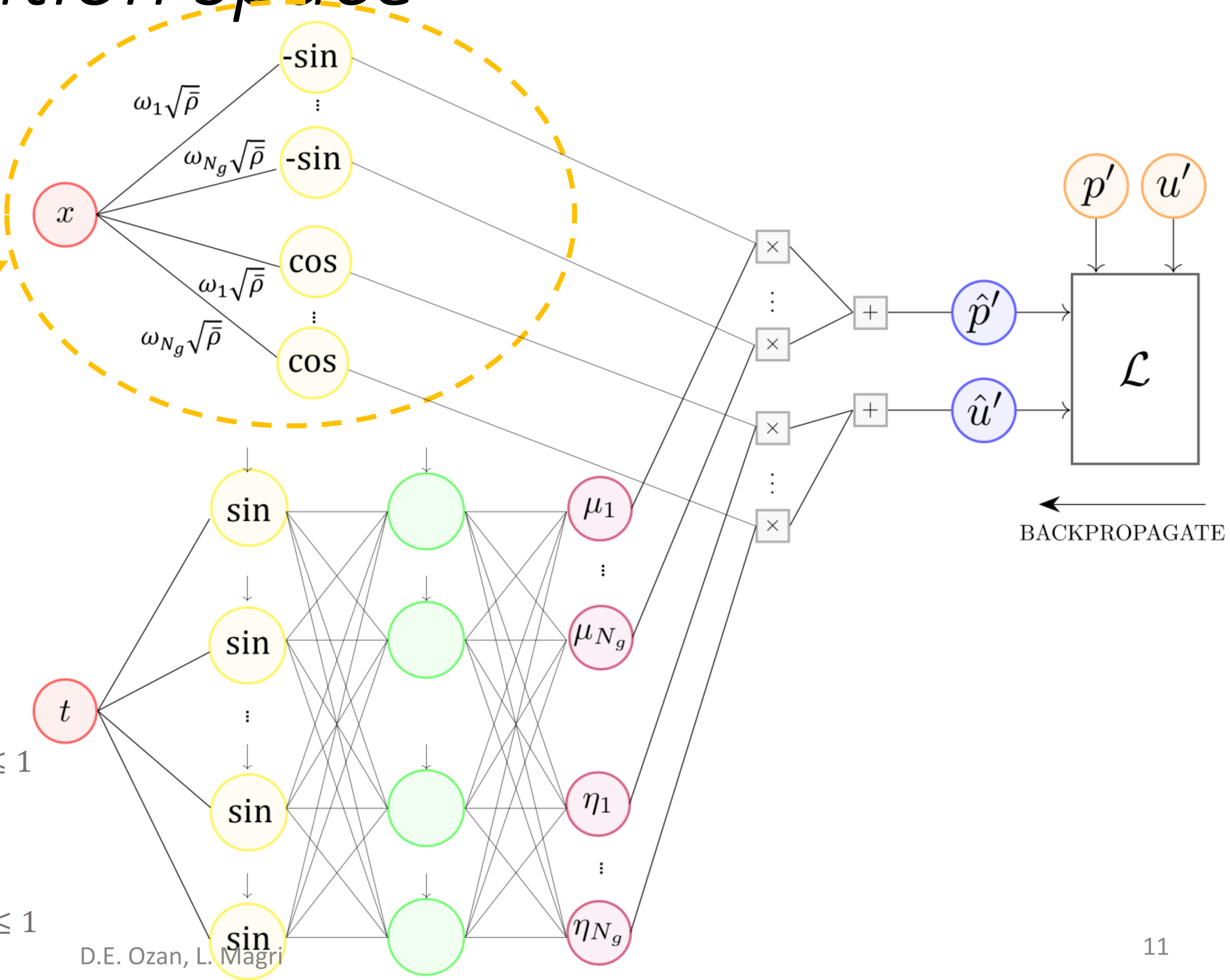
Choice of activation function





Thermoacoustic neural networks constrain properties of the solution space

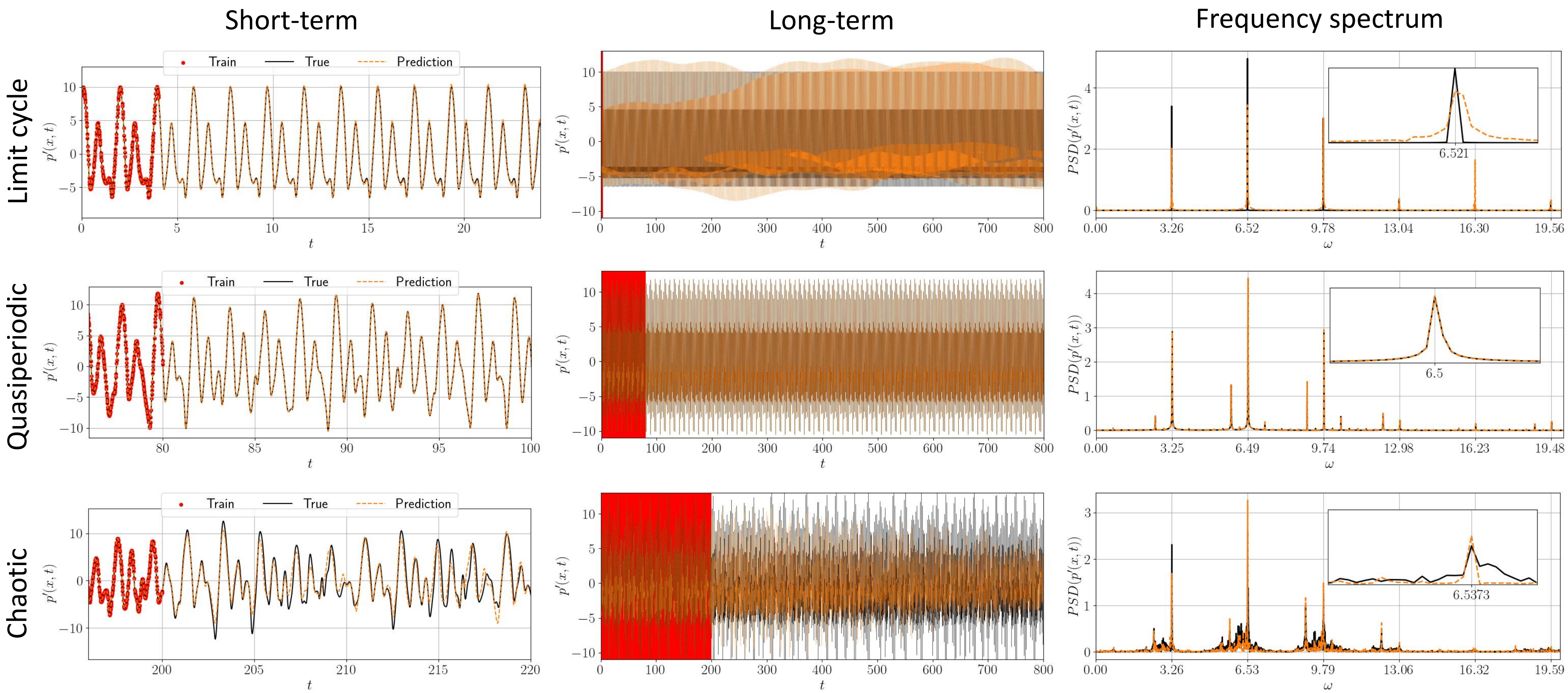




$$p'(x,t) = \sum_{j=1}^{N_g} \begin{cases} -\mu_j(t)\sin\left(\omega_j\sqrt{\bar{\rho}_1}x\right), & 0 \le x \le x_f \\ -\mu_j(t)\dfrac{\sin\gamma_j}{\sin\beta_j}\sin\left(\omega_j\sqrt{\bar{\rho}_2}(1-x)\right), & x_f \le x \le 1 \end{cases}$$

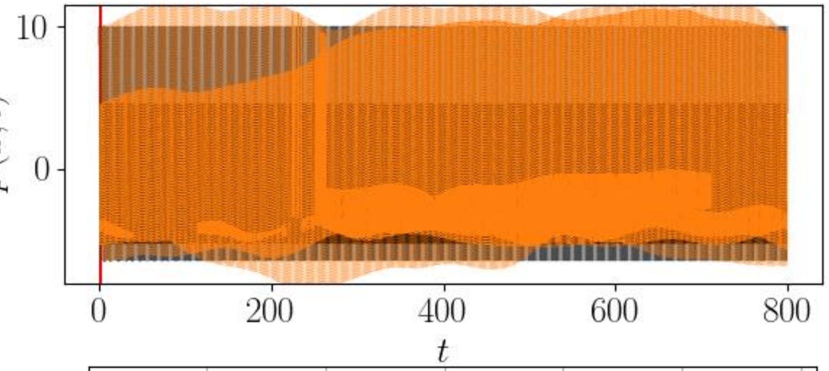
$$u'(x,t) = \sum_{j=1}^{N_g} \begin{cases} \eta_j(t)\cos\left(\omega_j\sqrt{\bar{\rho}_1}x\right), & 0 \le x \le x_f \\ -\eta_j(t)\dfrac{\sin\gamma_j}{\sin\beta_j}\cos\left(\omega_j\sqrt{\bar{\rho}_2}(1-x)\right), & x_f \le x \le 1 \end{cases}$$

Learning nonlinear regimes with *thermoacoustic neural network*

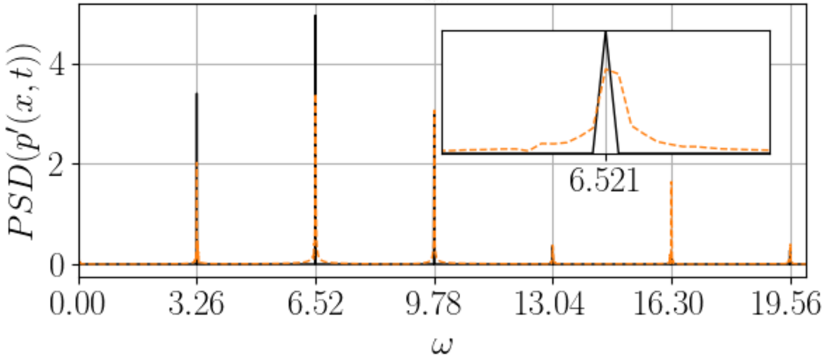


Constraining periodicity

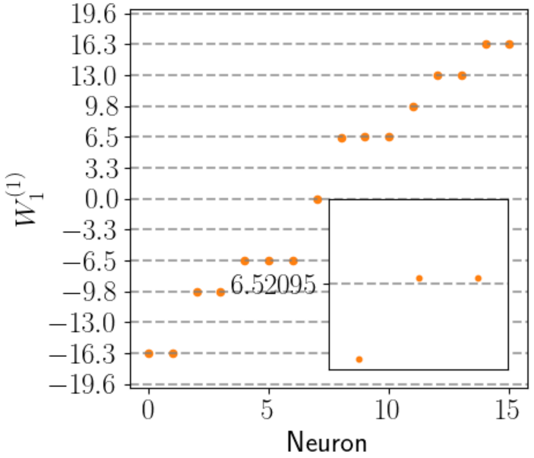
Long-term

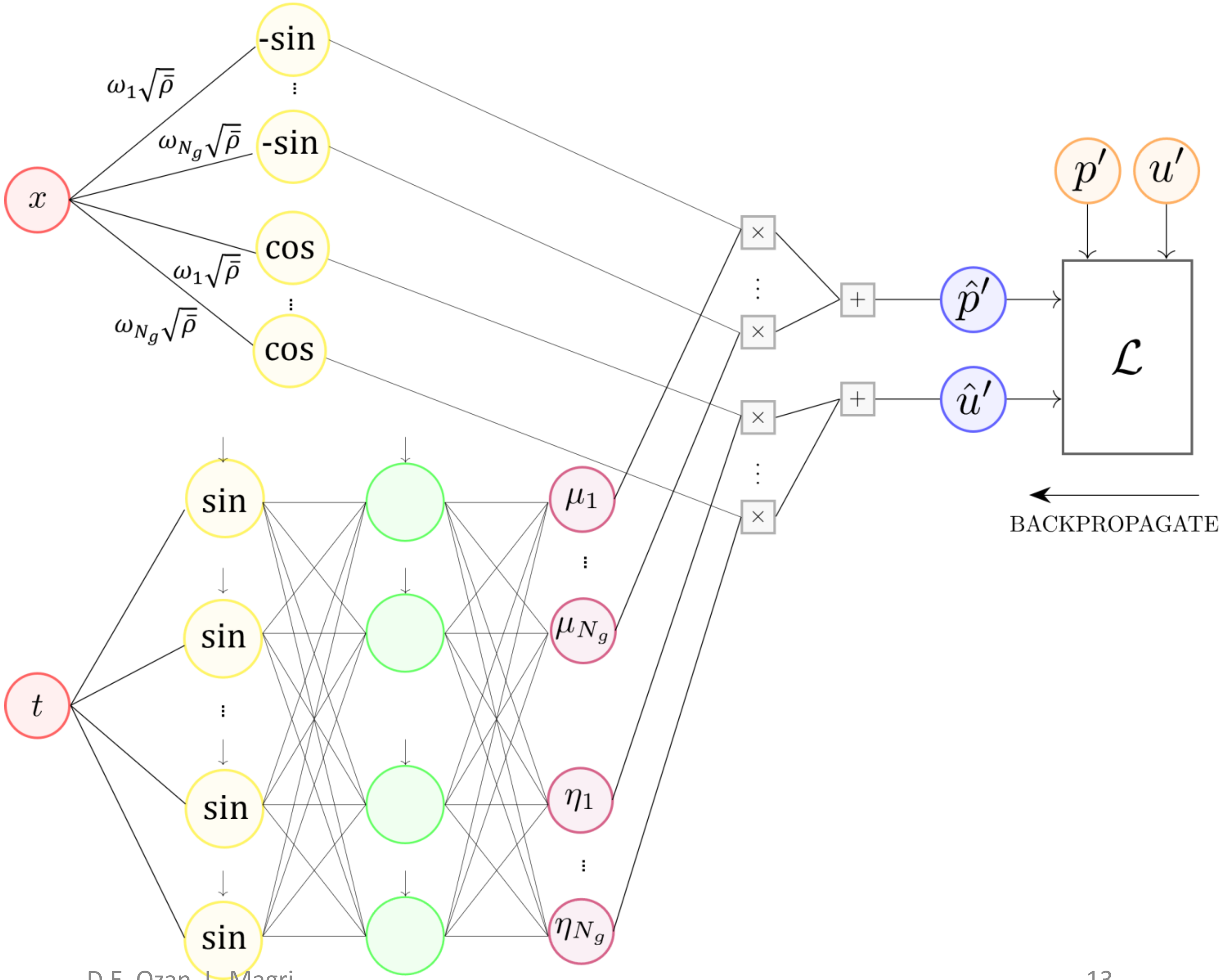


Frequency spectrum

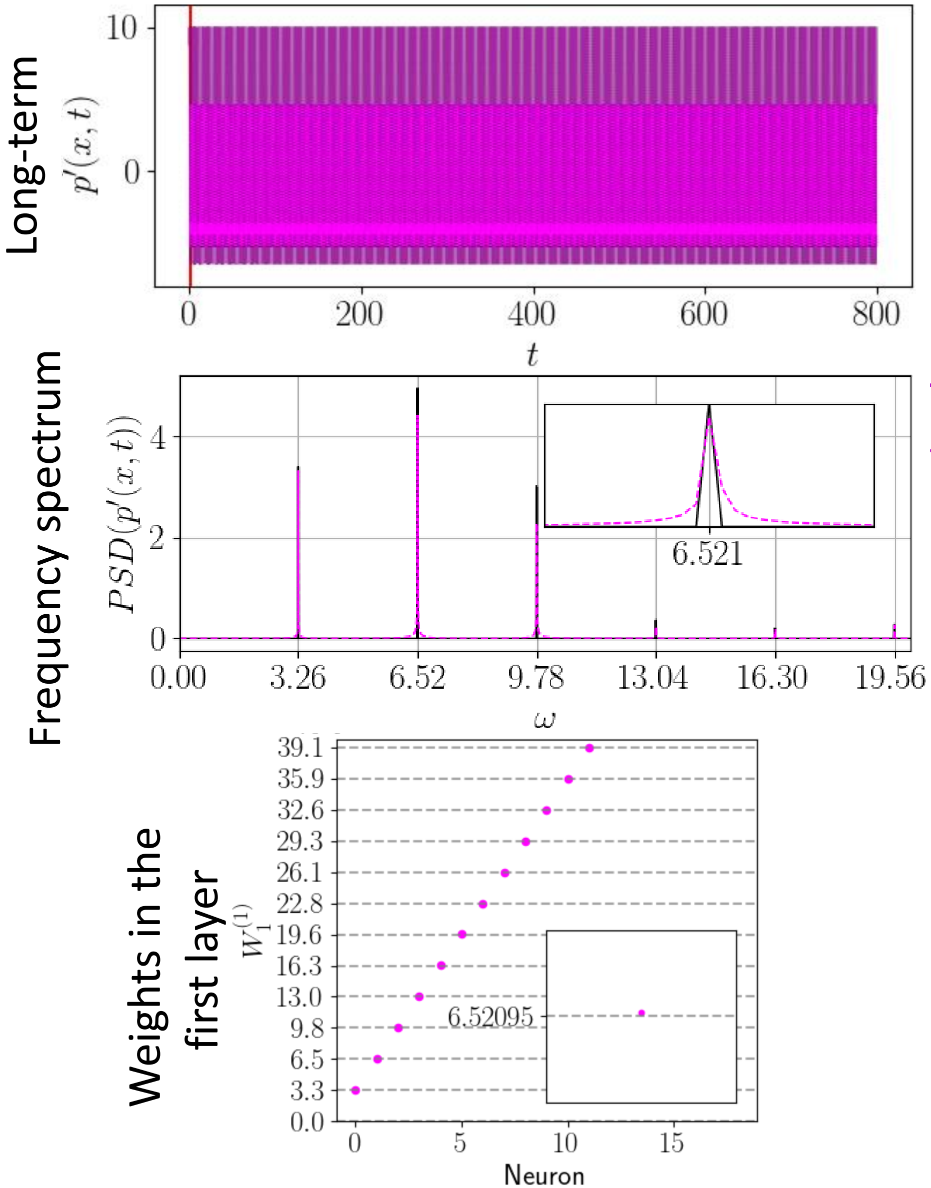


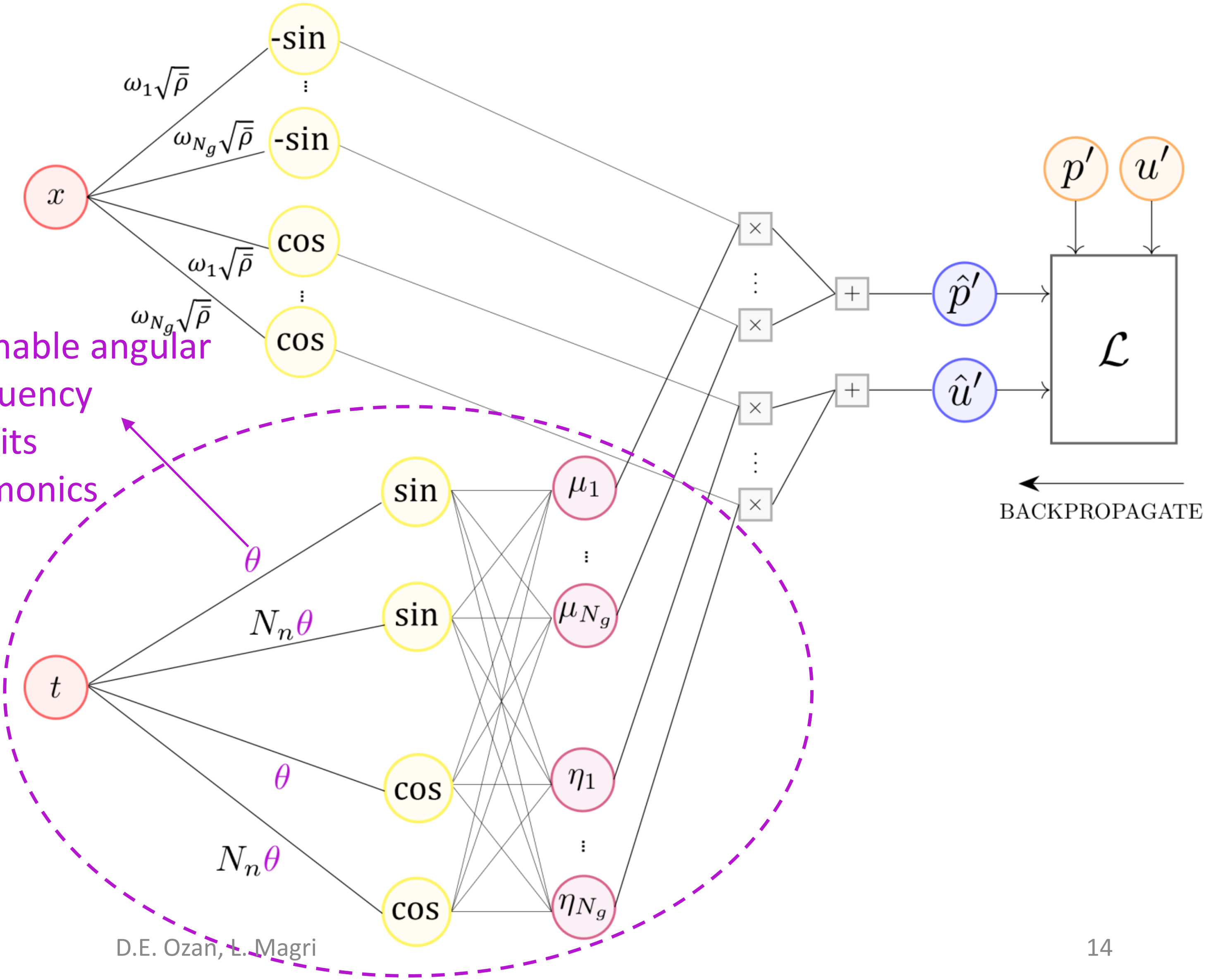
Weights in the first layer





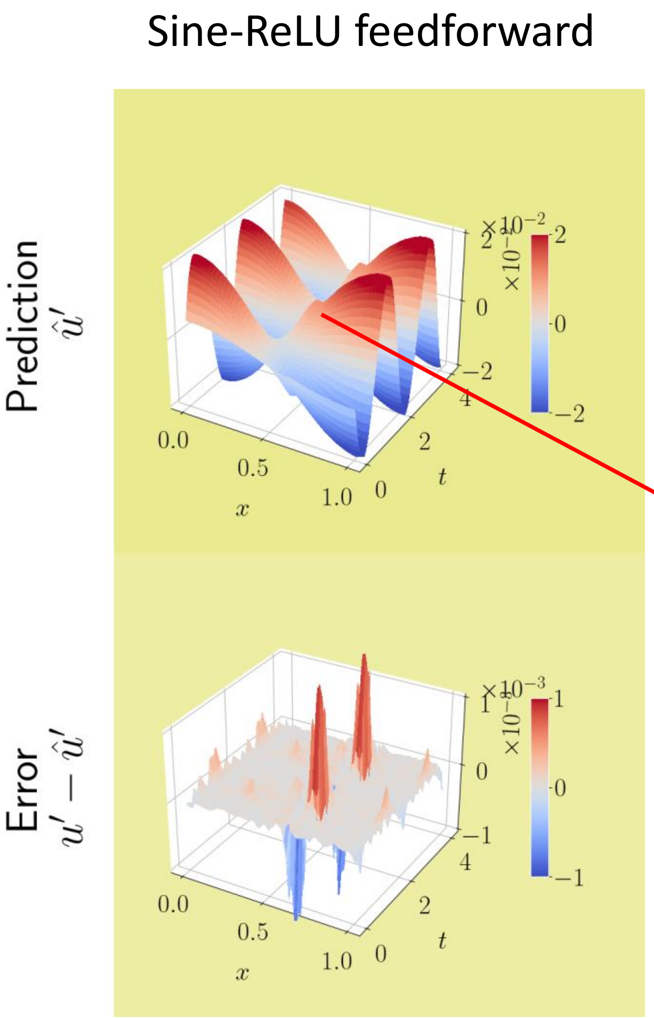
Constraining periodicity

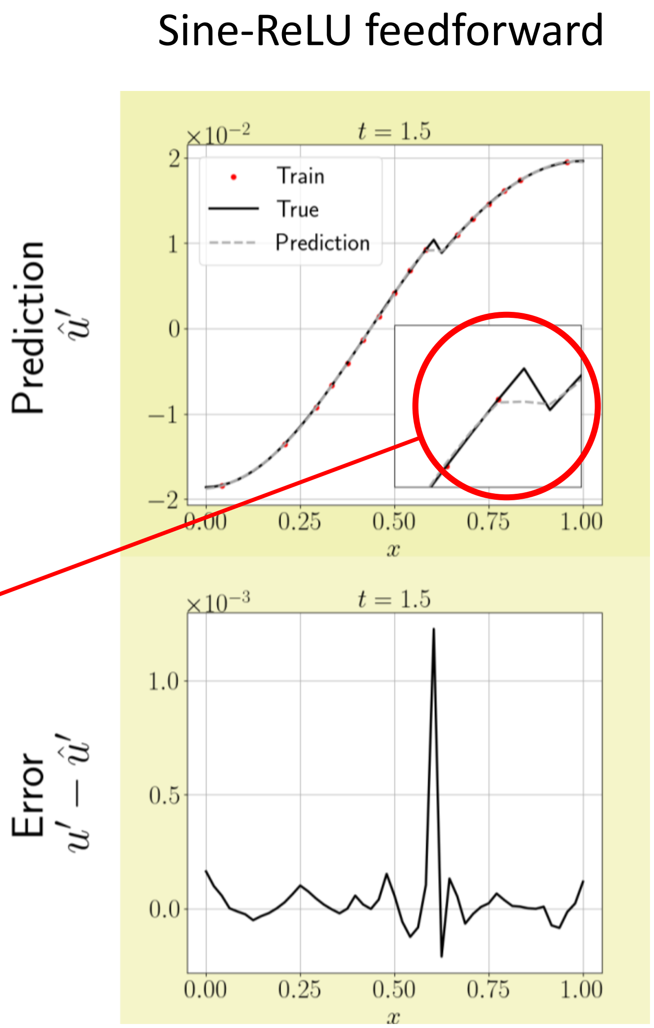




Results on higher fidelity data

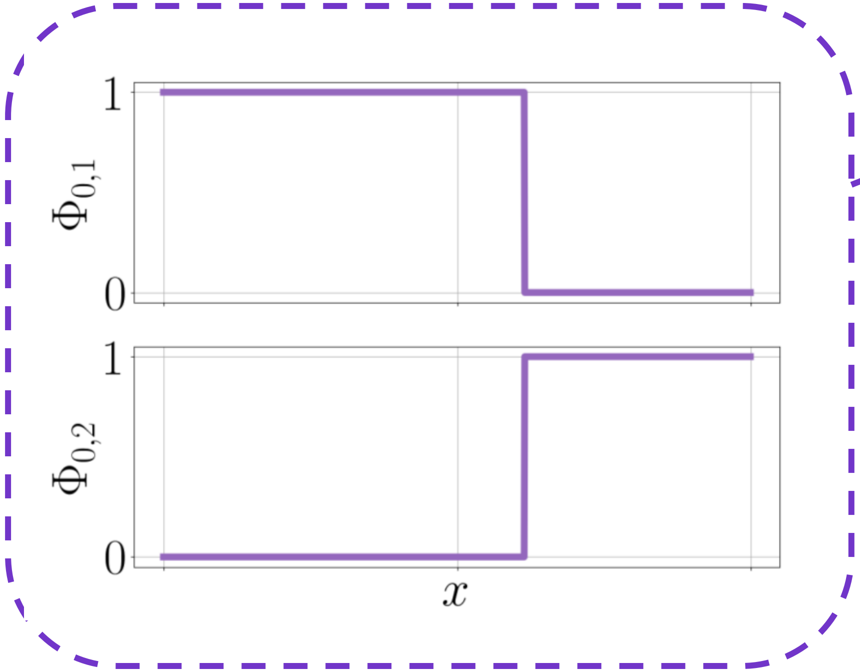
Periodic FNN can not capture discontinuity

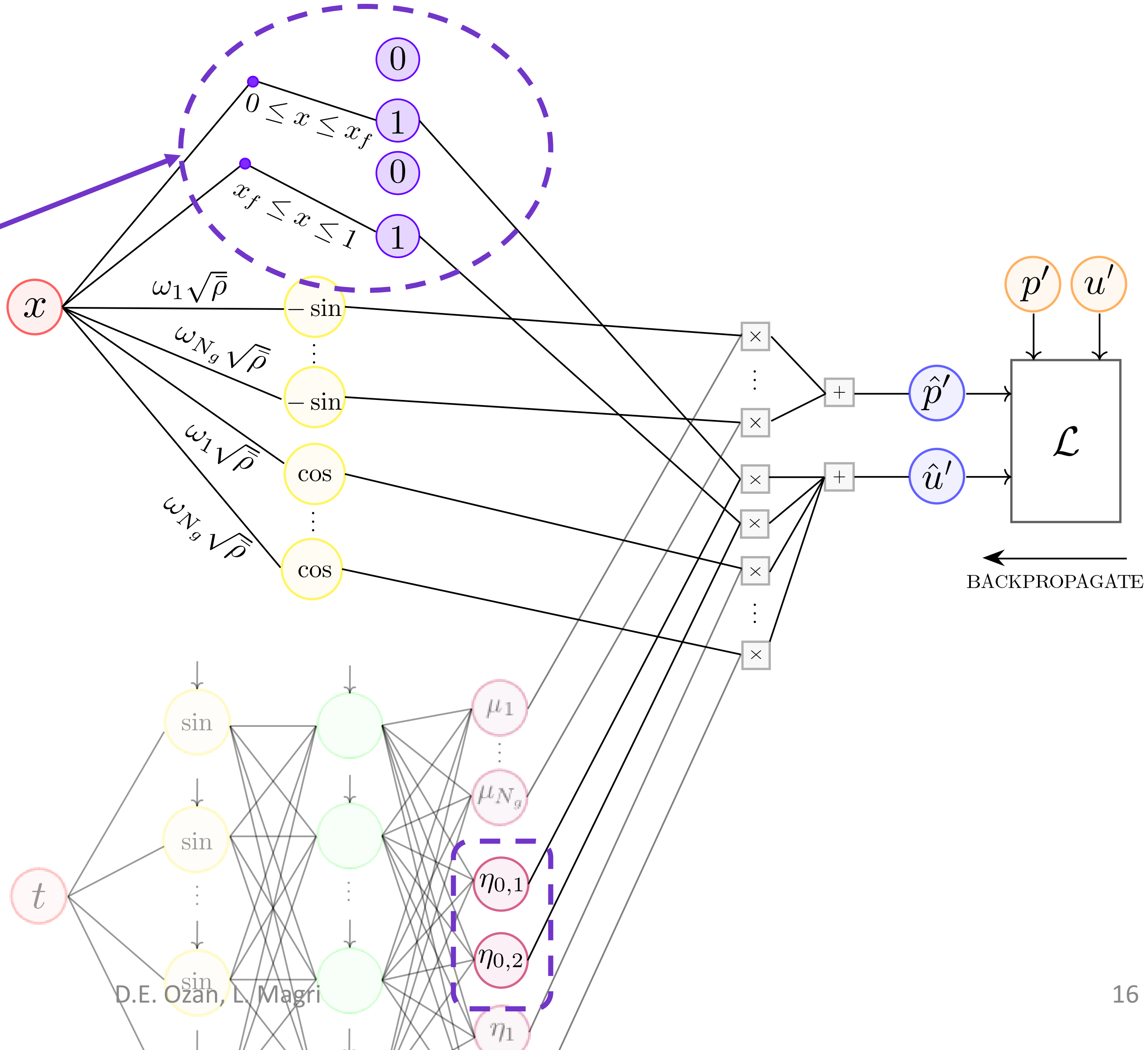




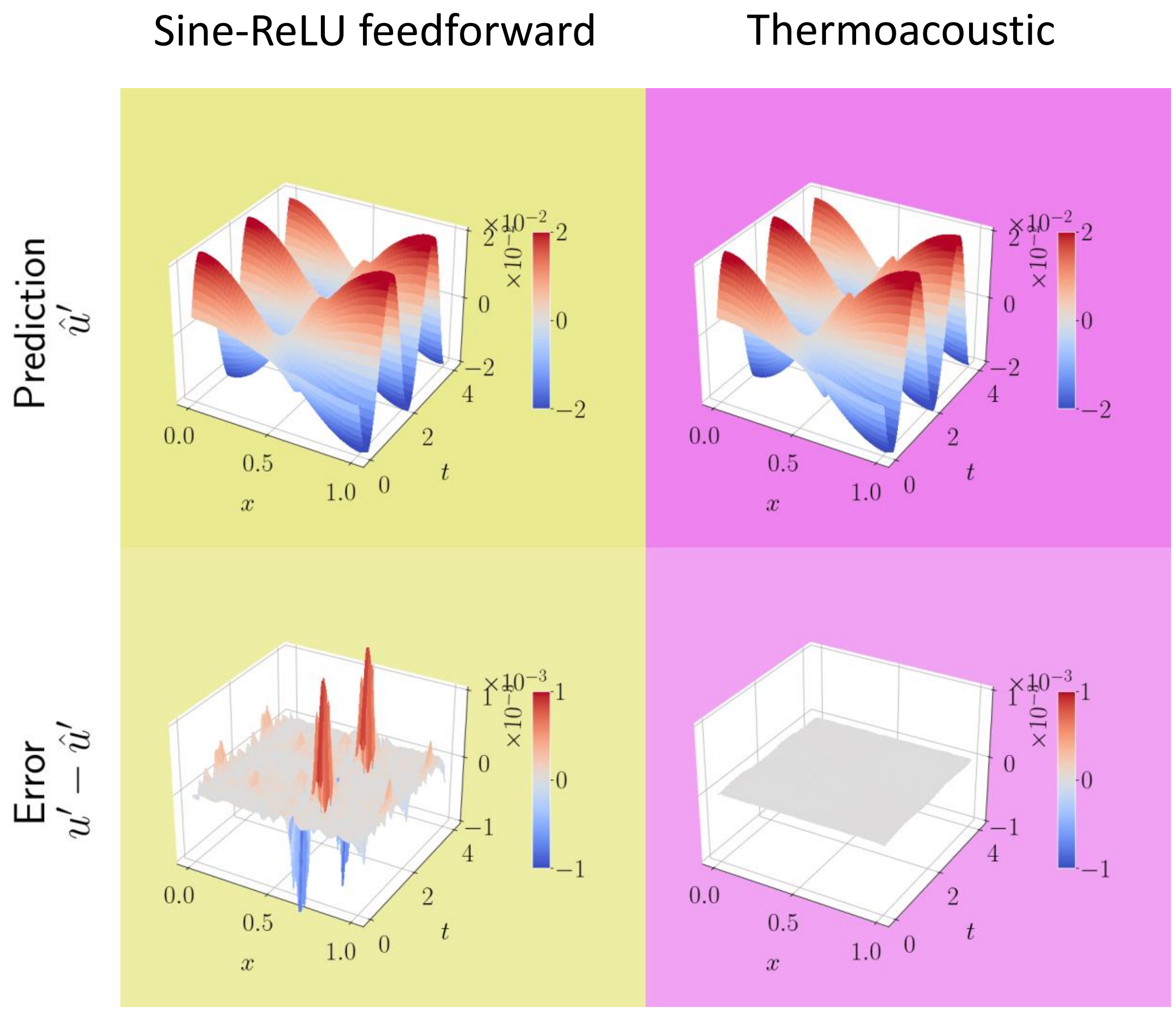
Jump discontinuity at the heat source location

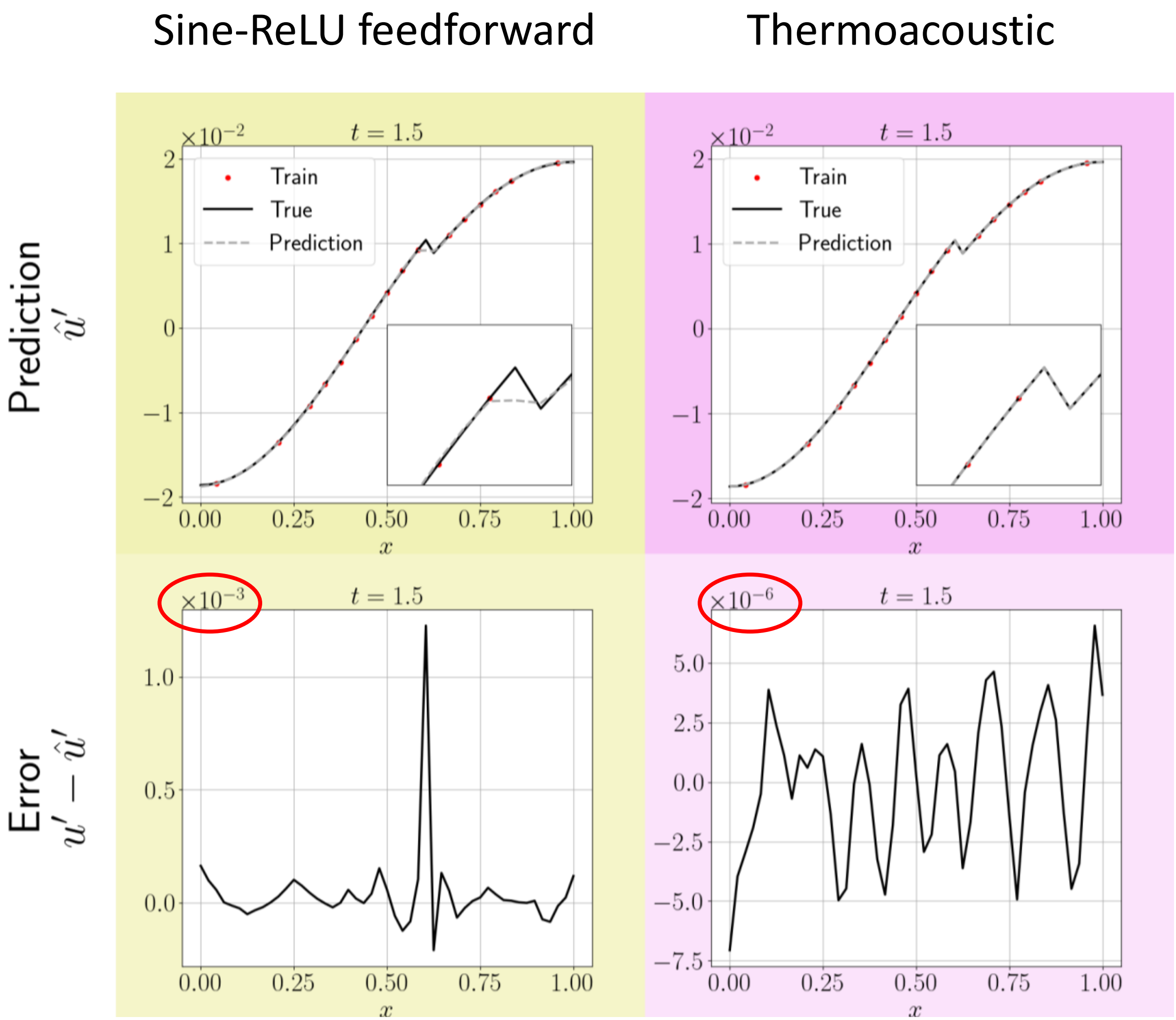
Handling the discontinuity via additional modes



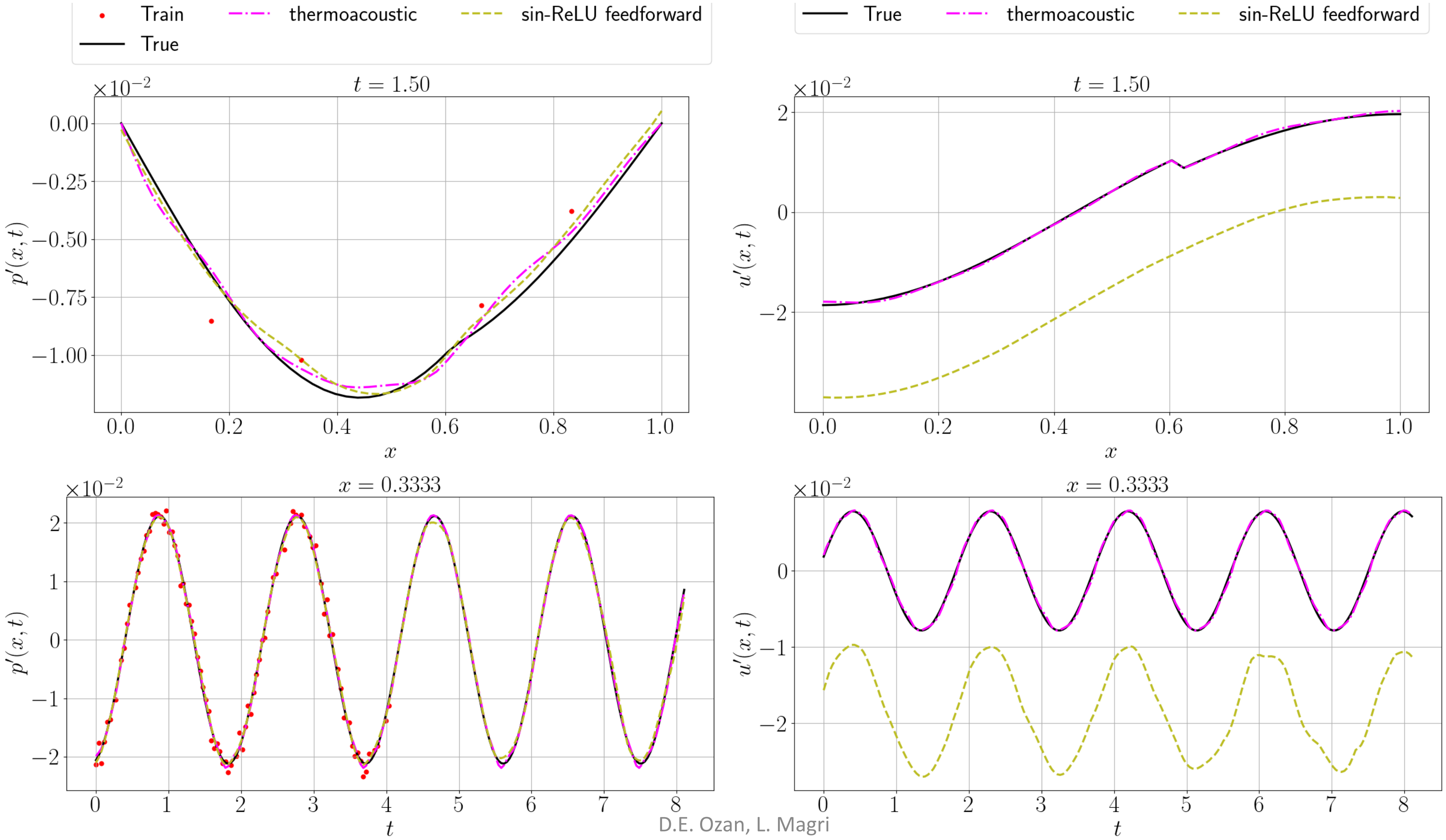


Results on higher fidelity data
Comparison of networks





Velocity reconstruction from pressure data



Conclusion

no constraints → hard-constrained

data-driven → physics-based

Feedforward NN	Sine activated FNN	Thermoacoustic NN	Periodic Thermoacoustic NN
+ flexible	+ extrapolates + requires less layers/neurons	+ hard-coded physics + interpretable + robust + requires less layers/neurons	+ fundamental frequency guaranteed
- big hyperparameter space - not robust - heavily relies on data	- performance limited by available data		- suitable for periodic series

+ we can add physics-information as a soft-constraint to any network